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CURRENT PAPERS AND DISCUSSIONS

		Discussion closes
Bridge and Tunnel Approaches. *John F. Curtin*	Nov., 1939	
Discussion	Mar., Apr., Dec., 1940	Closed
Analysis of Legal Concepts of Subflow and Percolating Waters. *C. F. Tolman and Amy C. Stipp*	Dec., 1939	
Discussion	Feb., Apr., May, June, 1940	Closed*
Miniature System of First-Order Alinement and Triangulation Control. *Floyd W. Hough*	Dec., 1939	
Discussion	Apr., June, 1940	Closed*
Pressure-Momentum Theory Applied to the Broad-Crested Weir. *H. A. Doeringsfeld and C. L. Barker*	Dec., 1939	
Discussion	Mar., Apr., June, 1940	Closed*
Chicago River Control Works. *H. P. Ramey*	Jan., 1940	
Discussion	June, Dec., 1940	Closed
Norris Dam Construction Cableways. *R. T. Colburn and L. A. Schmidt, Jr.*	Dec., 1939	
Discussion	Mar., Apr., June, Nov., Dec., 1940	Closed
Axioms in Roadway Soil Mechanics. *Henry C. Porter*	Feb., 1940	
Discussion	Sept., Oct., 1940	Closed*
Sealing the Lagoon Lining at Treasure Island with Salt. *Charles H. Lee*	Feb., 1940	
Discussion	June, Oct., Dec., 1940	Closed
Foundation Experiences, Tennessee Valley Authority: A Symposium	Mar., 1940	
Discussion	May, June, Sept., Dec., 1940	Closed*
Theory of Elastic Stability Applied to Structural Design. *Leon S. Moisseiff and Frederick Lienhard*	Jan., 1940	
Discussion	Nov., 1940	Dec., 1940
Design of Hinges and Articulations in Reinforced Concrete Structures. *George C. Ernst*	Apr., 1940	
Discussion	Oct., 1940	Dec., 1940
Permissible Composition and Concentration of Irrigation Water. *W. P. Kelley*	Apr., 1940	
Discussion	Sept., Nov., 1940	Dec., 1940
Progress Report of the Committee of the Structural Division on Oxygen Cutting (Flame Cutting) of Structural Steel	Apr., 1940	
Discussion	Sept., 1940	Dec., 1940
Masonry Dams: A Symposium	May, 1940	
Discussion	Sept., Oct., Nov., Dec., 1940	Dec., 1940
Model Tests, Bridge Pier Supported on Long Steel Piles. *Thomas F. Comber, Jr., and John M. Coan, Jr.*	June, 1940	Dec., 1940
The Practice of State Highway Departments in the Design of Abutments: Progress Report of a Special Subcommittee of the Committee of the Structural Division on Masonry and Reinforced Concrete	June, 1940	
Discussion	Oct., Dec., 1940	Dec., 1940
Concrete Specifications, Pt. 2	June, 1940	
Discussion	Nov., Dec., 1940	Dec., 1940
Maximum Probable Floods on Pennsylvania Streams. *Charles F. Ruff*	Sept., 1940	
Discussion	Dec., 1940	Jan., 1941
Superstructure of Theme Building of New York World's Fair. *Shortridge Hardesty and Alfred Hedefine*	Sept., 1940	Jan., 1941
The Grand Central Terminal in Perspective. *William J. Wilgus*	Oct., 1940	
Discussion	Dec., 1940	Feb., 1941
Earthquake Stresses in the San Francisco-Oakland Bay Bridge. *Norman C. Raab and Howard C. Wood*	Oct., 1940	
Discussion	Dec., 1940	Feb., 1941
Transatlantic Seaplane Base, Baltimore, Maryland. *W. Watters Pagon*	Oct., 1940	Feb., 1941
Consolidation of Embankment and Foundation Materials: Progress Report of Sub-Committee No. 2 of the Committee of the Soil Mechanics and Foundations Division on Earth Dams and Embankments	Oct., 1940	
An Investigation of Steel Rigid Frames. *Inge Lyse and W. E. Black*	Nov., 1940	Mar., 1941
Reliability of Station-Year Rainfall-Frequency Determinations. *Katharine Clarke-Hafstad*	Nov., 1940	Mar., 1941
Cavitation in Outlet Conduits of High Dams. *Harold A. Thomas and Emil P. Schuleen*	Nov., 1940	Mar., 1941

NOTE.—The closing dates herein published are final except when names of prospective discussers are registered for special extension of time.

* Publication of closing discussion pending.

CONTENTS FOR DECEMBER, 1940

PAPERS

DISCUSSIONS

CONTENTS FOR DECEMBER, 1940 (*Continued*)

INDEXES

For Index to all Papers, the discussion of which is current in PROCEEDINGS, *see page 2*

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

FORT PECK SLIDE

By T. A. Middlebrooks,[1] Assoc. M. Am. Soc. C. E.

Synopsis

Tests and analyses conducted on the soils and rocks encountered at the Fort Peck Dam (in northeastern Montana, on the Missouri River) which have a bearing on the slide that occurred on September 22, 1938, are the basis of this paper. Since very little information has been published on the investigations conducted prior to the slide, the first part of the paper has been devoted to that subject. The major part is devoted specifically to the studies conducted after the slide, to determine its cause and to outline a method of repair.

Introduction

After preliminary investigations were completed, it was realized that, in addition to the difficulties normally expected from the construction of a full hydraulic fill dam of this magnitude, the foundation conditions were complicated (see Fig. 1). A thick plastic clay stratum approximately 60 ft below the natural ground surface extended over the center two thirds of the valley and well outside the upstream and downstream toes of the dam. Overlying, underneath, and outside the limits of the clay stratum, there were stratified pervious materials varying from fine sand to gravel. At each abutment the shale was either exposed or lay under a relatively shallow overburden. An extensive program of foundation exploration and laboratory investigations was planned to obtain the necessary information for design purposes. A soil testing laboratory was established and equipped to perform all types of soil tests, including the determination of photoelastic stress distribution. In addition to the regular shear, consolidation, and permeability tests, numerous special tests and experiments were performed, such as: (1) Expansion tests on shales and tills; (2) large-scale shear and consolidation tests; (3) clay model tests to observe action during plastic deformation; (4) photoelastic tests on gelatin models to develop a rational means of computing the stress distribution in the embankment and foundation; (5) earth-dam models of the main dam on approximate scales of 1 to 50 and 1 to 200 and earth models of dike sections on a scale of 1 to 2; and

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **April 15, 1941.**

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(6) sand models to obtain information on the effect of consolidation of the core of the shells.

In order to keep a close check on the settlement and seepage during and after construction, settlement plates and perforated pipes were installed in the

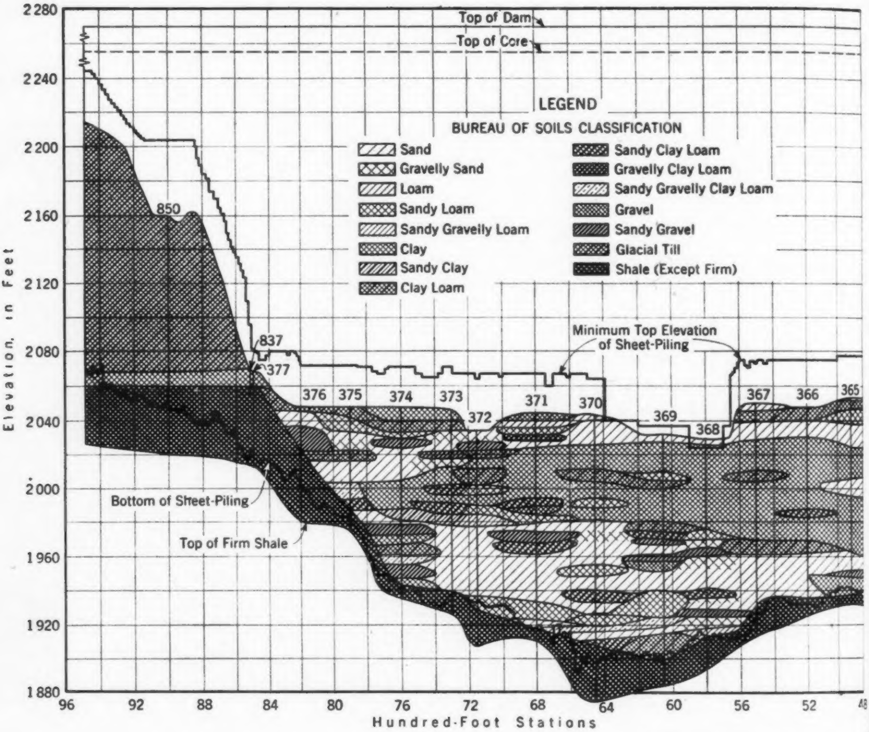


FIG. 1.—SOIL PROFILE ALON

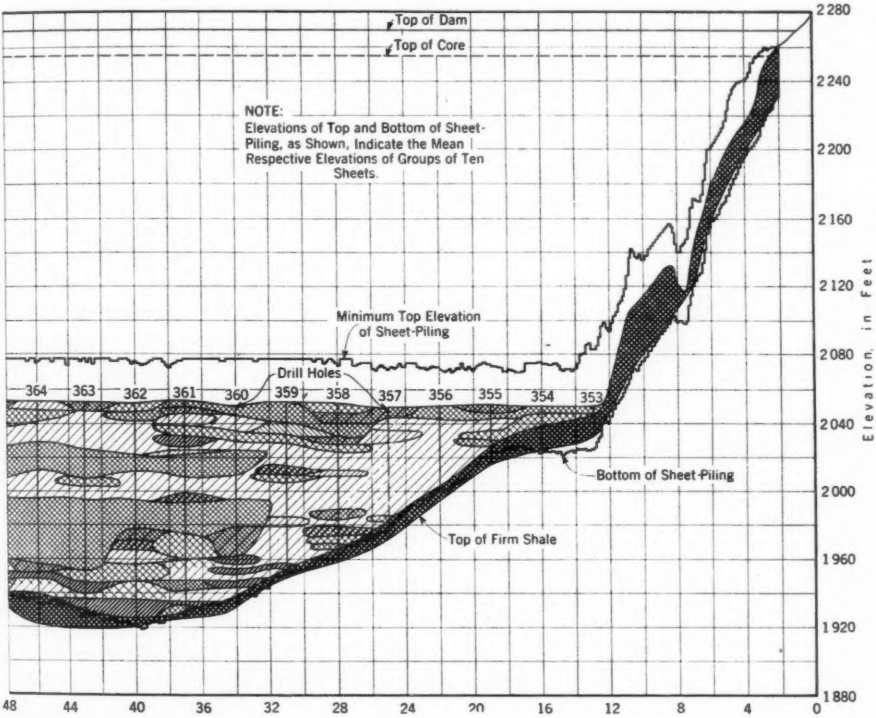
fill on several cross sections of both the dam and dike sections. In addition, hub stakes were set outside both toes of the dam and on the surface of the completed part of the slopes. The elevation and location of these hub stakes and plates were determined periodically. They were usually read monthly, but in some cases critical locations were observed daily.

TEST AND ANALYSIS PRIOR TO SLIDE

Investigations were conducted prior to and during construction to insure a safe structure with reference to the following possible sources of trouble: (*a*) Bursting of the shell due to excessive core pressure; (*b*) slides in the shells due to excessive deposition of core material and clay balls; (*c*) plastic flow of the foundation clay; (*d*) slides in the foundation sands or silts; and (*e*) slides on the shale surface.

"Bursting of Shell."—The possibility of the core bursting the upstream shell was investigated prior to and checked during construction, using the method

proposed by Glennon Gilboy,[2] Assoc. M. Am. Soc. C. E., in 1934. It was found that the dam was safe even when the core was assumed to be a liquid, weighing 100 lb per cu ft. The upstream core and outer slope were 1 on $\frac{2}{3}$ and 1 on 4, respectively. Special care was taken in placing this fill to insure that the core



CUTOFF WALL, FORT PECK DAM

did not encroach upon the shell. Sand lenses were allowed to protrude into, but not through, the core, and they were found to be most helpful in accelerating the core consolidation. Shear, consolidation, and permeability tests were run on undisturbed samples taken from the core during construction. Density samples were taken every 5 ft in elevation through the core on each 500-ft station at the center line every three months in order to check closely the amount and rate of consolidation. These tests indicated that the core material consolidated at an exceedingly rapid rate.

"Slides in Shell."—Consideration was given to the fact that slides in the shells might occur from two conditions: (1) Deposition of core material on the beaches due to wide pools; and (2) deposition of large pockets of clay lumps from the pocket discharge lines. Daily control samples, as well as permanent record samples, at fixed intervals were taken to check on the core limits and

[2] "Mechanics of Hydraulic-Fill Dams," by Glennon Gilboy, *Journal*, Boston Soc. of Civ. Engrs., July, 1934, p. 185.

the type of material being placed in both the core and the shells. It was found necessary to establish maximum and minimum core limits, since it was obvious that a fixed limit was not practical in construction. Upstream, the allowable maximum limit was only a short distance outside the minimum, and the construction forces were required to keep the pool as close to the minimum limits as possible. When the trap-line lifts were completed before the pool was pushed back to the minimum limits, ground lines were used to push the pool back to the required limits. To allow more flexibility of operation, wider pool variations were permitted downstream where the outer slope was 1 on $8\frac{1}{2}$.

To establish a sound basis on which to fix the control requirements for the allowable quantity of clay to be deposited on the shells from the core, a complete series of tests were run to determine the shearing strength of various mixtures of sand and clay. Tests were performed on sand mixed with 0 to 30% clay by dry weight. The shearing strength decreased at a fairly uniform rate until approximately 25% clay had been added, at which point the voids of the sand were completely filled with clay and the shearing strength rapidly approached that of clay. Using the results of these tests, specifications were written to restrict the clay content in the shells such that the coefficient of friction would be maintained at more than 0.6. This was the value necessary to prevent bursting of the shell due to core pressure assuming no consolidation. The tests showed that the clay content could not exceed approximately 8.0%. Actually, the shell. were constructed throughout with less than 3% clay.

Deposition of clay lumps on the beaches resulted from the excavation of stiff clay in the borrow pits. It was impracticable to prevent the pumping of clay lumps entirely, since there were numerous small clay strata in the borrow pits that could not be avoided. To exert satisfactory control over the deposition of clay lumps in the fill, it was necessary to determine the type of borrow-pit clays that formed clay lumps so that these strata could be avoided wherever possible, and it was necessary to run shear and permeability tests to determine the quantity of clay lumps that would not be detrimental to the structure.

Samples of clay lumps actually pumped by the dredges were analyzed and the results compared directly with samples from borrow pits. It was found that most of the clay lumps were derived from clay strata classified as medium (50% to 70%) clay, and fat (70% to 100%) clay. Dredging was then restricted to borrow pits containing a minimum quantity of these types of clay.

"*Plastic Flow of Foundation Clay.*"—After the extent of the plastic clay in the foundation had been outlined by the preliminary explorations, an extensive investigation was made to check the stability of the proposed sections both for the completed dam and for the 1-on-3 construction slopes in the closure section.[3] The special foundation exploration consisted of ten 6-in. borings through the clay strata. Continuous undisturbed samples were taken for the full thickness of the clay strata, and shear and consolidation tests were run on representative samples from each hole. Since considerable deformation occurs in a clay prior to ultimate failure in shear, it was feared that sufficient deforma-

[3] "Foundation Investigation of Fort Peck Dam Closure Section," by T. A. Middlebrooks, *Paper G-3*, *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Cambridge, Mass., 1936.

tion might occur in the thick plastic clay stratum (60 ft maximum thickness) to cause failure in the dam before the full strength of the clay could be mobilized. After a careful study of this condition in clay models constructed in the laboratory, it was decided the shearing strength should be taken at the "yield point" instead of the point of "ultimate failure." The result was a decrease in the deformation of approximately 75% from the ultimate without decreasing the strength more than approximately 25%. Strength of the clay was determined from consolidated and unconsolidated shear tests by taking into consideration the percentage of consolidation during construction.

Since failure of clay would occur due to plastic flow, the Swedish slide method[4] for determining the stability was not considered applicable. The elastic-theory method was used for determining the stress distribution in the clay strata. The works of J. Boussinesq,[5] S. D. Carothers,[6] Leo Jürgenson,[7] and others were utilized to develop a rational method for determining these stresses. Photoelastic experiments on a gelatin model were performed to check the mathematical application of the elastic theory to this type of structure. As a result of this investigation,[8] the construction slopes in the closure section were flattened from 1 on 3 to 1 on 5, and a berm was added upstream for the full length of the clay stratum, Station 30+00 to Station 70+00, Fig. 1.

"Slides in Foundation Sands and Silts."—During the investigation of the clay stratum sufficient undisturbed samples of the foundation sands and silts were obtained to determine their strength. It was found that there was ample factor of safety against failure of these materials.

"Slides on Shale."—Explorations in the right abutment disclosed that beneath several feet of exposed disintegrated shale, there was a zone of blocky weathered shale approximately 30 ft thick. There was a gradual transition from the blocky weathered shale into subfirm and firm shale. In the valley adjacent to the abutment a pervious overburden averaging 30 ft in thickness overlays a thin stratum of clay 1.5 ft thick. This clay rested directly on subfirm shale.

The possibility of failure in the shale in this vicinity was considered as highly improbable, since subfirm shale of similar classification excavated in other locations on the project had shown that the subfirm shale and the bentonite seams contained therein possessed ample strength to prevent failure. Detail investigation of this zone after the slide showed that the upper part of stratum classified as subfirm shale was actually weathered shale. It was also found that the clay stratum adjacent to the abutment would consolidate 100% during construction. For this reason, all shear tests on the clay were performed on consolidated samples. Stability analyses made on the basis of the results

[4] "Erdstatische berechnungen mit reibung und kohäsion (adhäsion) und unter annahme kreiszlindrischer gleitflächen," by W. Fellenius, W. Ernst (publisher), Berlin, 1927.

[5] "Application des potentiels à l'étude de l'équilibre et du mouvement des solides élastiques, principalement au calcul des déformations et des pressions que produisent, dans ces solides, des efforts quelconques exercés sur une petite partie de leur surface ou de leur intérieur; mémoire suivi de notes étendues sur divers points de physique mathématique et d'analyse," by J. Boussinesq, *Mémoires de la Société des Sciences, de l'Agriculture et des Arts de Lille*, Vol. 13, 1885, p. 1.

[6] "The Elastic Equivalence of Statically Equipollent Loads," by S. D. Carothers, *Proceedings of the International Mathematical Congress*, Vol. II, Toronto, Canada, 1924, p. 519.

[7] "The application of Theories of Elasticity and Plasticity to Foundation Problems," by Leo Jürgenson, *Journal*, Boston Soc. of Civ. Engrs., July, 1934, p. 206.

[8] "Foundation Investigation of Fort Peck Dam Closure Section," by T. A. Middlebrooks, *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., June, 1936.

obtained from the tests indicated that the dam was safe against failure from this source.

Test and Analysis After Slide

Directly after the slide occurred near the right abutment on September 22, 1938 (see Fig. 2), all previous test data and analyses were reviewed to assist in arriving at the most logical probable causes of the slide. Preliminary in-

Fig. 2.—Aerial Photograph

vestigations indicated that the slide may have been caused by one or a combination of the following conditions: (1) Movement along some weak zone in the shale; (2) movement along the shale surface; (3) bursting of the shell due to excessive core pressure; or (4) temporary liquefaction of the shell or foundation sand, or both.

To determine the extent of additional laboratory work necessary for a complete investigation, tests were run by the latest approved methods to check

previous testing, so that there would be no unnecessary duplication. The standard direct shear apparatus was used for most of the testing, but the shearing strength of the core and the critical density of the shell material were determined by the latest developed triaxial shear machine.

Movement in Shale.—In order to investigate possible movement along some weak zone in the shale, undisturbed samples were taken from test pits, drift,

Dam Immediately After Slide

and large-diameter (12-in. to 36-in.) core borings. A considerable number of small (3-in. to 6-in.) borings were made through the slide material and overburden and 40 ft into the shale to outline the distribution of the various materials. Continuous drive samples were taken of the slide material and overburden and cored samples from the shale. Undisturbed samples of clay, weathered shale, and bentonite from the test pits, drifts, and large-diameter borings were tested for shear and consolidation characteristics in the laboratory.

2

In addition, large-scale field tests were run on undisturbed weathered bentonite in the drifts driven into the weathered shale. Representative results of these tests are shown in Fig. 3 which also shows the required strength to give a safety factor of 1.0 for the section prior to failure. It will be noted that most of the tests on bentonite show a strength below the required strength line, whereas most of the tests on clays and weathered shale show a strength above that required. It will also be noted that the strength of the core material is well above that required, the coefficient of friction averaging about 0.6.

Results of these tests indicate clearly that the bentonite and weathered shale were the weakest materials in the foundation, and that undoubtedly the weathered bentonite was by far the weakest material encountered. This conclusion is substantiated by all the explorations. Numerous movement zones or slide planes were found in the weathered shale by the drive samples taken from the small borings. In practically all cases bentonite was mixed with gouge material in these zones of movement. In some cases, the large-diameter cores showed that bentonite seams had been completely removed by the slide; some were thoroughly mixed with shale along a gouge zone, others remained as definite seams but were drawn out showing considerable movement.

Observations made on holes drilled into the shale revealed another very important factor that was difficult to take into consideration in the analyses but which, it is reasonable to assume, had a major effect on stability. Holes drilled into the shale downstream of the slide and on the west abutment showed that there was a high hydrostatic pressure in the shale, which was considerably in excess of the maximum lake level or the saturation line in the dam. In Table 1, which shows the water levels obtained in the various drill holes, it

TABLE 1.—HYDROSTATIC PRESSURE IN FOUNDATION SHALE
(Elevations, in Feet Above Sea Level)

Location of Hole		Natural ground	Embankment	Saturation line	Equivalent column[b]	Location of Hole		Natural ground	Embankment	Saturation line	Equivalent column[b]
Station	Range[a]					Station	Range[a]				
4.92	5.16U	2,175	2,259	....	2,195	76.02	2.81U	2,046	2,211	2,105	2,183
11.25	3.750U	2,100	2,189	2,155	2,175	78.94	0.50U	2,048	2,252	2,220	2,310
11.30	6.67U	2,078	2,175	2,155	2,145	79.01	2.80U	2,042	2,216	2,115	2,256
13.50	8.92U	2,046	2,156	2,115	2,115	80.00	0.37D	2,048	2,247	2,220	2,310
13.75	3.75U	2,046	2,183	2,162	2,118	81.77	2.78U	2,041	2,219	2,113	2,213
16.25	3.75U	2,044	2,182	2,174	2,074	81.94	0.50U	2,048	2,251	2,220	2,232
18.75	8.50U	2,048	2,158	2,122	2,170	84.94	0.50U	2,073	2,250	2,220	2,260
13.50	1.10D	2,060	2,185	2,154	2,128	84.98	2.72U	2,057	2,217	2,115	2,190
15.00	4.00D	2,049	2,232	2,125	2,210	87.94	0.50U	2,156	2,250	....	2,100
20.00	4.00D	2,046	2,229	2,127	2,226	90.93	0.50U	2,176	2,250	2,219	2,104
75.94	0.50U	2,048	2,250	2,220	2,242	93.94	0.50U	2,214	2,250	2,220	2,181

[a] U = upstream; D = downstream. [b] Elevation of a column equal to hydrostatic pressure.

will be seen that one hole developed a pressure head equal to that of a water column extending to El. 2,310, approximately 60 ft above the maximum height of the fill and 90 ft above the saturation line. Other holes developed pressures of more than 100 ft above the lake level or the saturation line in the dam. Borings prior to the construction of the dam showed no artesian water in the

shale. All artesian wells in this vicinity are drilled through the Bearpaw shale to a depth of from 600 to 1,000 ft, and none of these wells developed a pressure even closely approximating those obtained from these later holes drilled through the dam.

The most logical reason for the existence of such a high hydrostatic pressure in the shale is that trapped water was squeezed out of the shale, due to the superimposed load of the dam, most likely along weathered bentonite seams.

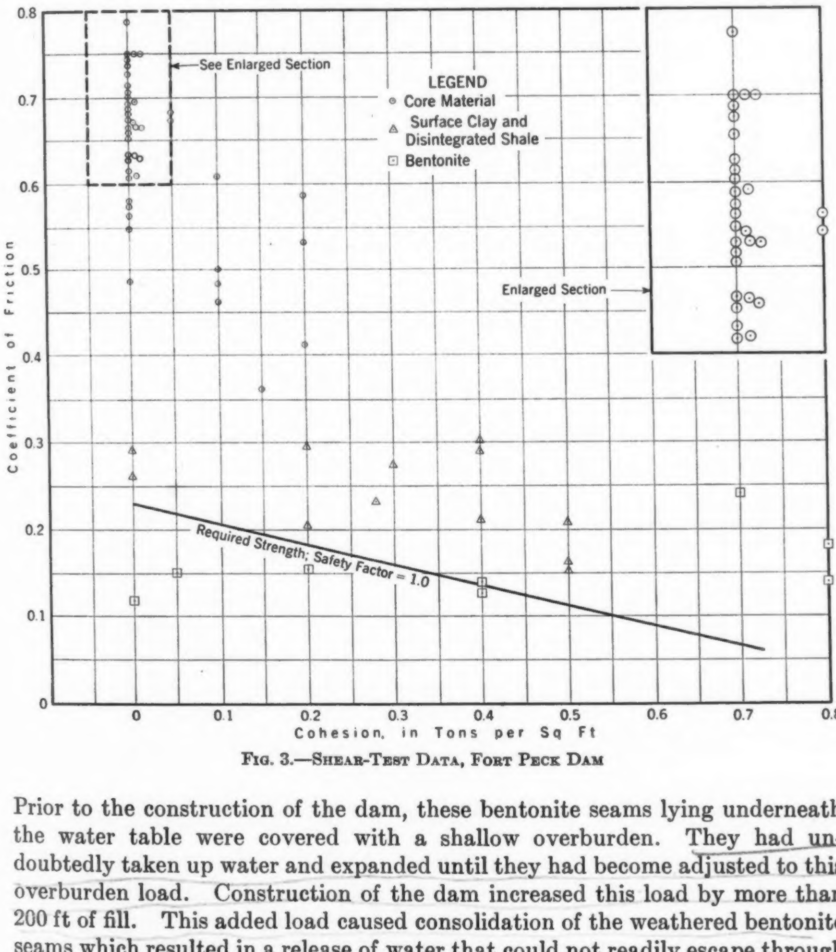


FIG. 3.—SHEAR-TEST DATA, FORT PECK DAM

Prior to the construction of the dam, these bentonite seams lying underneath the water table were covered with a shallow overburden. They had undoubtedly taken up water and expanded until they had become adjusted to this overburden load. Construction of the dam increased this load by more than 200 ft of fill. This added load caused consolidation of the weathered bentonite seams which resulted in a release of water that could not readily escape through the overlying impervious shale. It was trapped along the seams, developing the high hydrostatic pressures which were later observed. It is the firm conviction of the writer that this excess hydrostatic pressure which was found to exist in the shale was one of the major causes of the slide and accounts in large part for the speed at which, and the distance to which, it moved.

Movement on Shale.—In the slide area there was approximately 1.5 ft of clay and disintegrated shale overlaying the subfirm shale. This clay was in direct contact with a free draining foundation sand. Additional tests confirmed earlier investigations and showed that the thin clay strata would consolidate rapidly with the application of load. Consolidated shear tests were used for determining the shearing strength of this material. All of the strengths so determined were well above the required strength to give a factor of safety of 1.0.

Bursting of Shell.—In view of the wide core and relatively flat core slopes, bursting of the shell due to excessive core pressure was considered by many in

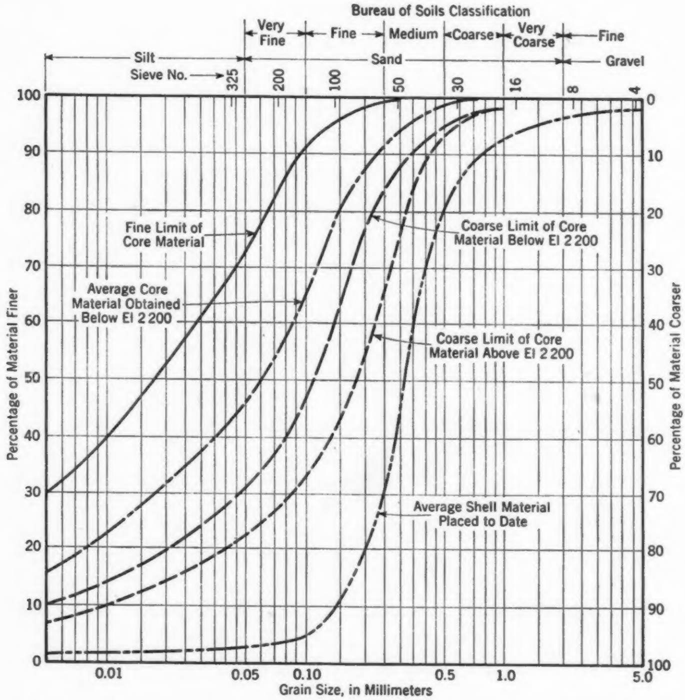


FIG. 4.—MECHANICAL ANALYSES FOR CORE-MATERIAL LIMITS AND THE ACTUAL MATERIAL IN THE FILL

the early stages of the slide investigation to be the most logical reason for the slide. Therefore, a comprehensive testing program was outlined to determine the strength of the core and shell materials. Shear tests were performed on the direct shear machine supplementing the earlier tests. In addition, triaxial tests were run at Harvard University, Cambridge, Mass., at the U. S. Waterways Experiment Station, Vicksburg, Miss., and in the laboratory of the Fort Peck District. Results of some of these tests on the core material are shown in Fig. 3. The coefficient of friction of the core material averaged approximately 0.6, with only a small degree of cohesion. The triaxial tests showed the

core material to be even stronger than the direct shear tests indicated. After watching some triaxial tests run and studying the results, one engineer remarked that "the core material appeared to brace itself against failure." It was concluded from all the tests that the core material was practically as strong as the shell material.

An attempt was made to measure the horizontal pressure exerted by the core by the use of Goldbeck earth pressure cells attached to a long pile. The high degree of consolidation of the core material made it impossible to set the pile in the core without impairing the accuracy of the cells.

Composition of the average core material is shown in Fig. 4. Limits of the core as actually constructed did not vary greatly from the design limits, except for sand lenses which extended into, but not through, the core. Consolidation of the core was accelerated by these lenses of sand which protruded into the core from the transition zone. Fig. 5 shows the degree of consolidation for core material on the center line for different elevations of the fill. These data were obtained about two months prior to the slide. The tests comprise samples between Station 15+00 to 85+00, along the axis. The void ratios (averages plotted in Fig. 5) were determined volumetrically from drive samples 2 in. by 18 in., taken in July, 1938. The core pool varied from El. 2,213 to 2,200.

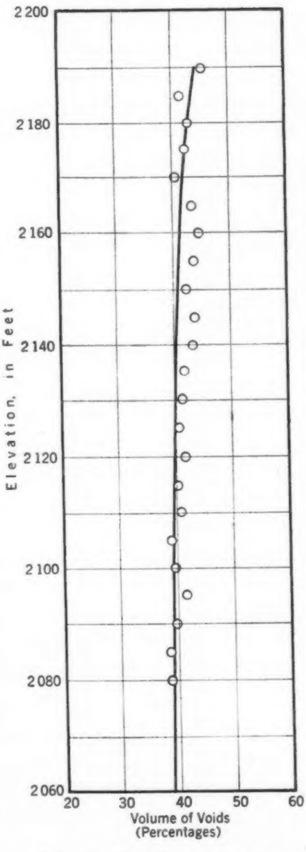


Fig. 5.—Core Consolidation

The relatively high degree of consolidation and strength of the core, as shown by the tests, was further borne out by the action of the material during the slide as shown by the topography and exploration after the slide. The core remaining in place on the west end of the slide stood on very steep slopes and exhibited high resistance to scour from the core pool water. Borings showed that the downstream part of the core did not move out with the slide, but remained intact while the overlying shell material moved upstream over it. Boring No. 4 at Station 14+75, approximately on the axis, showed that a large hole had developed during the slide down to natural surface. Core material did not squeeze into it as might have been expected, but it was filled with sand.

The upstream toe trestle was pushed out ahead of the slide. If the shells had burst due to core pressure, the most likely path of failure would have been over the trestle. Failure did not occur where the core was the least consolidated, which was the closure section. This part of the dam was raised

approximately 150 ft in four months, whereas the dam at the east abutment was raised to this height over a period of twenty-eight months.

Therefore, it was concluded from the test data, explorations, and surface observations made directly after the slide that the core was decidedly stronger than anticipated in the original design and consequently possessed a good safety factor against bursting the shells.

Temporary Liquefaction of Sand.—"Liquefaction" of the sand shell or foundation sand (or both) was advanced as another possible cause of the slide. The fact that loose sand, when subjected to a shock or change in stress, will slough or slump has been recognized by engineers for ages past. In 1936 Arthur Casagrande, Assoc. M. Am. Soc. C. E., showed a method whereby the density required to prevent sloughing for different sands and different conditions can be approximated.[9] It has been designated as the "critical density" and is defined as the density at which no change in volume will occur at a given vertical load. Any material having a density greater than the critical will

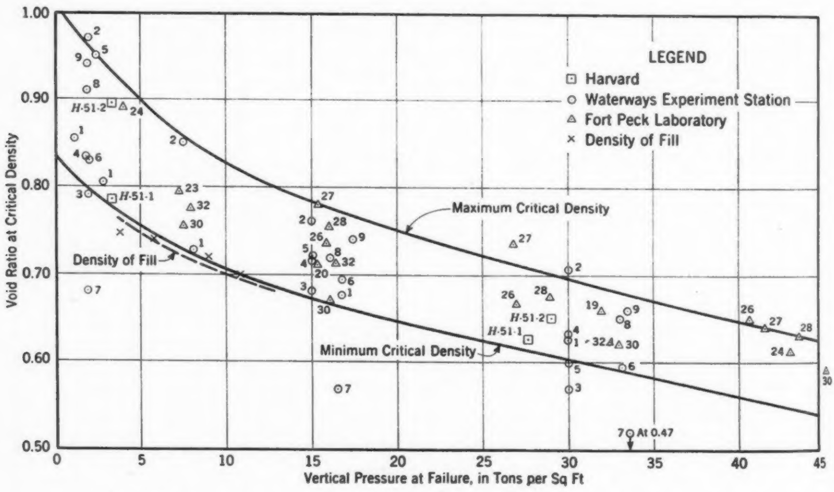


FIG. 6.—SUMMARY OF CRITICAL DENSITY TESTS ON SHELL MATERIAL

tend to expand when subjected to shearing stresses; therefore, this material when saturated will brace itself against failure; whereas, a material looser than the critical will consolidate when subjected to shearing stresses, thereby allowing the load of a saturated material to be partly carried by the water which reduces its shearing strength.

It has been concluded that a material denser than the critical is definitely safe against flow. On the other hand, it does not follow that all materials looser than the critical density are unsafe. This looser material will definitely have some reserve strength, depending on its density. This problem did not enter into the slide investigation since it was found that density of the shell

[9] "Characteristics of Cohesionless Soils Affecting the Stability of Slopes and Earth Fills," by Arthur Casagrande, *Journal*, Boston Soc. of Civ. Engrs., January, 1936, p. 13.

material was equal to, or greater than, the critical. It is a problem, however, that deserves careful consideration by investigators in the future.

In order to determine the actual density of the shell material and to obtain samples for tests, several test pits were excavated in the undisturbed upstream shell. Continuous 6-in. undisturbed samples were taken, and the density and mechanical composition determined. Critical density tests using the triaxial machine were run on representative samples at Harvard University, U. S. Waterways Experiment Station, and Fort Peck. Results of these tests are shown in Fig. 6. Density of the Fort Peck shell material is shown as a heavy broken line. These tests show that the hydraulic fill is denser than required. In addition to the critical density tests on disturbed shell materials shown in Fig. 5, tests on undisturbed samples to determine the effect of stratification were run in the Fort Peck Laboratory. These test results showed that there was an even greater margin of safety against flow of the shell material than was indicated by the other tests.

The conclusions drawn from the test data were substantiated by inspection of the undisturbed 36-in. cores. The inspection disclosed that definite stratification, as it existed in the hydraulic fill, was undisturbed by the movement except for numerous shear planes. This stratification might not be destroyed by streamlined flow of the shell material, but it can be stated definitely that there would be no shear planes in the sand if flow had occurred. Therefore, it can be concluded from the tests and explorations that the shell material was of sufficient density to prevent it from becoming liquefied even under the severe stress conditions that occurred during the slide.

The distance that the slide moved upstream may have been due to some liquefaction of the slide materials, as has been concluded by the Board of Consultants. It is the opinion of the writer that the evidence proves such a liquefaction must have occurred along the bentonite seams in the shale, especially after sufficient movement had occurred to mix the bentonite with the trapped water. The inlet channel, cut approximately 800 ft from the toe of the dam, also had a major influence on the distance that the slide moved. The cut is approximately 500 ft wide and 30 ft deep (bottom elevation, 2,030.0).

General Questions

Before proceeding to the stability analysis, it seems desirable to discuss briefly a number of questions raised during the investigation which, directly or indirectly, affect any stability analyses that might be made. The questions are listed below and discussed in subsequent paragraphs:

(1) What effect did filling and lowering of the reservoir have on the slide?

(2) Was the construction rate too rapid?

(3) Did excessive settlement occur in the slide area prior to the slide?

(4) Did seepage away from the abutment through the fill have any appreciable effect on the slide?

(5) Was the core at the east abutment finer or coarser than the remaining portion of the dam?

(6) What effect did the slide have on the adjacent abutment and dam?

Effect of Reservoir Water.—At the time of the slide the reservoir was at El. 2,117.5, having been drawn down from El. 2,137.0 in approximately 60 days. The shale and bentonite in which the slide occurred were below the natural water table and were therefore saturated before the dam was constructed. Since the load of the dam was much greater than the reservoir pressure, the water was actually being squeezed out of the foundation. The water in the reservoir, therefore, could not enter the shale and bentonite to weaken it. The reservoir water at El. 2,117.5 decreased the stresses in the foundation from what they would have been if it had been empty. This decrease in stress, however, was in part counterbalanced by the increase in stress due to the drawdown; but, on the whole, the water in the reservoir had very little effect on the stability of the structure. Although the effect was small, these conditions were given full consideration in the stability analyses.

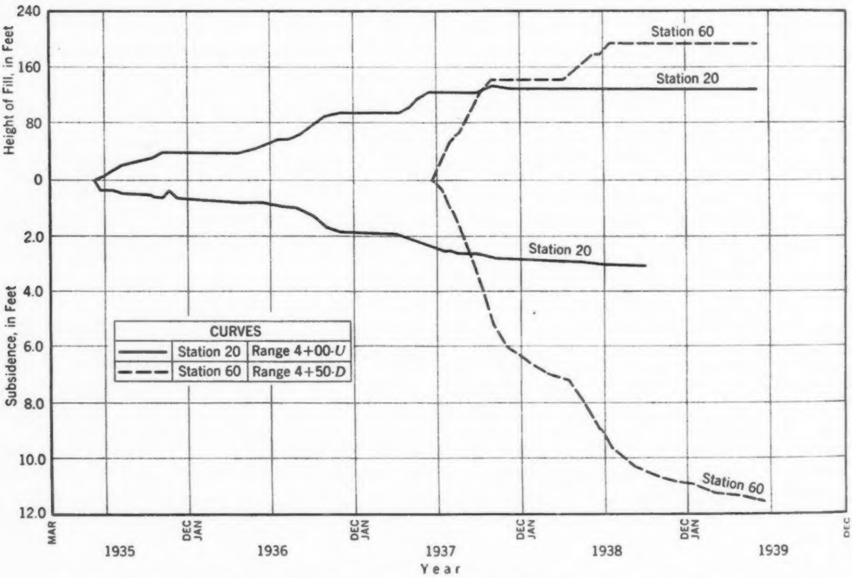


FIG. 7.—RECORD OF FOUNDATION SUBSIDENCE AND FILL HEIGHT, STATIONS 20+00 AND 60+00

Rate of Construction.—The rate of construction for the part of the dam in the slide was comparatively slow. The rate of construction is shown as a curve in Fig. 7 in connection with the subsidence. It will be noted that the fill was brought up in stages. The first five months of construction brought the fill up approximately 40 ft, followed by a winter rest period of five and one half months. Then in seven months the fill was raised approximately 55 ft, and another rest period of five months occurred. This was followed by about two months of operation, prior to closure, in which the fill was raised approximately 30 ft. Closure operations allowed this fill to rest another four months, after which approximately 15 ft of fill was placed in one month. Following a rest period of five months, the fill was then raised approximately

60 ft in the five months prior to the slide. Increase in the fill height averaged approximately 5 ft per month including the shutdown periods. The maximum rise per month in any one period was approximately 15 ft.

In the slide area the construction rate was extremely slow compared with the closure section. The fill in the closure section rose 160 ft in four months, rested five months, and then rose 60 ft more in five months. The average rise per month was nearly 15 ft and the maximum was 40 ft. Therefore, if trouble was to be experienced in the embankment due to rapid construction, surely it would have occurred in the closure section.

Any reasonable extension of the construction period could not have been expected to increase materially the strength of the weathered shale or bentonite or to release the pressure in the relatively impervious shale where the source of the trouble existed.

Settlement.—The question concerning excessive settlement can be easily answered by reference to Fig. 7. It will be noted that the settlement at a point about the middle of the upstream slope had practically stopped. The settlement in this area amounted to only about 2% of the fill height, whereas in the closure section where the plastic clay is approximately 60 ft thick, the settlement amounted to more than 10 ft, or more than 5% of the fill height. Settlement in the slide area occurred at a rate of less than 0.08 ft per month; but in the closure section this rate was approximately 0.6 ft per month. Obviously then, it is most unlikely that excessive settlement contributed to the failure.

Seepage.—Seepage moving outward from the abutment undoubtedly raised the saturation line in the dam slightly, but the additional stress added to the foundation was negligible when compared with the total stress set up by the embankment.

Texture of Core Material.—During the preliminary stages of the investigation some engineers thought that the core was composed of finer material at the abutment and that greater core pressure was exerted there. Actually the major portion of the core was coarser at the abutments, a condition which it was impractical to prevent.

Effect of Slide on Adjacent Abutment and Dam.—The slide had no material effect on either the abutment or the dam outside the slide area. A small surface slough occurred in the abutment due to the toe support being removed quickly. However, it was of no importance.

Stability Analyses

Stability analyses were made, using the Fort Peck elastic theory method[3] and the static slide method.[4]

Section Prior to Slide.—Several cross sections were analyzed in the slide area. Station 15+00 has been selected because it is a representative cross section and because the slide started at approximately this location. Figs. 8 to 10 show the stability analyses of the section by the elastic theory method, prior to failure. Distribution of shearing stresses along the rigid boundary was computed and plotted in Fig. 8. The effect of seepage forces, the differ-

ence in weight of shell and core material, and the elevation of the lake were taken into consideration in determining the stress distribution. All fill materials were assumed to be homogeneous and isotropic, and had the following weights:

Material	Weight (lb per cu ft)
Sand material above the saturation line	105
Sand material below the saturation line	60
Core material (buoyed)	65
Disturbed shale (buoyed)	78

Using values of $\tan \phi = 0.185$ (ϕ = angle of internal friction) and cohesion $C = 0.2$ tons per sq ft, the shearing strength of the shale (broken line in Fig. 8) was plotted so that a direct comparison could be made with the shearing stress. It will be noted that there was an overstressed zone from range 4+00 to 6+25. Failure of the weathered shale and bentonite undoubtedly started in this overstressed zone and gradually increased until a sufficiently large area had been overstressed to produce failure of the entire structure. This is borne out by statements of the eye witnesses concerning the fact that the shell moved nearly straight out and the core pool and adjacent shells moved down as if dropping into a hole.

Attention is directed to the fact that in this analysis no consideration is given to the strength of the fill material. Where the weak material is in the foundation, failure starts in this material and progresses back into the embankment. It follows, therefore, that failure of the fill material in this case is incidental to the prior failure of the foundation material. This is one of the fundamental weaknesses of any static slide method, since progressive failure is not taken into consideration.

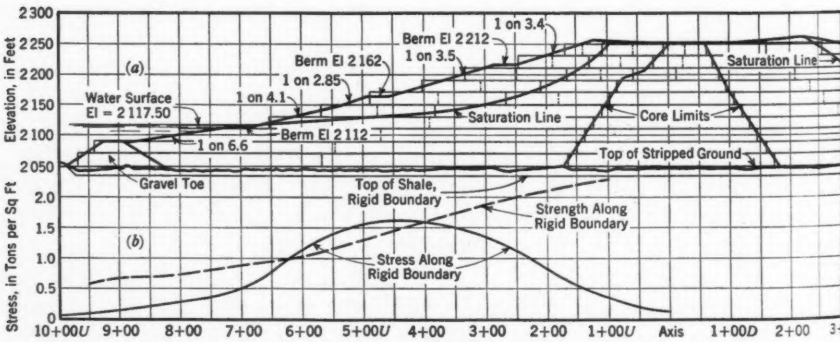


FIG. 8.—STRESS AND STRENGTH ($\tan \phi = 0.185$, $C = 0.2$ TON PER SQ FT) ALONG RIGID BOUNDARY

In this method of analysis the over-all safety factor (Fig. 9) is computed by taking the ratio of the accumulated shearing strength and stress by starting at the toe (Fig. 8) and progressing back to a point where it becomes a minimum. It will be noted in Fig. 10 that the safety factor drops below 1.0 at approximately range 4+00 and remains below to approximately range 2+00.

Attention is also directed to the fact that the test (Fig. 9) data, except for the weathered shale and bentonite, lie above the required strength line for a safety factor of 1.0.

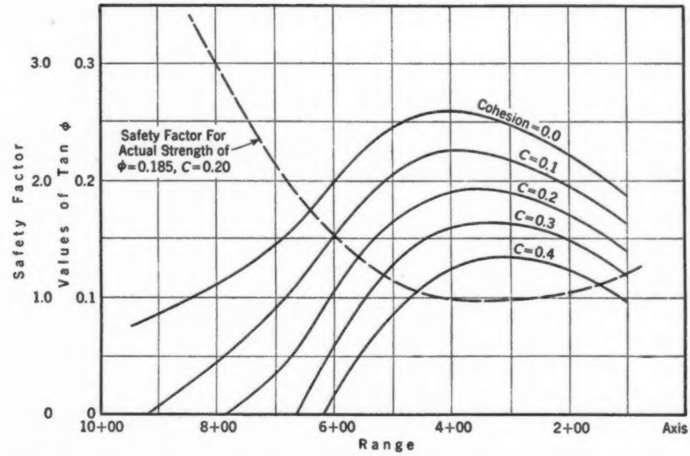


FIG. 9.—RELATION OF COHESION AND TAN ϕ ALONG THE RIGID BOUNDARY FOR A SAFETY FACTOR OF 1.0

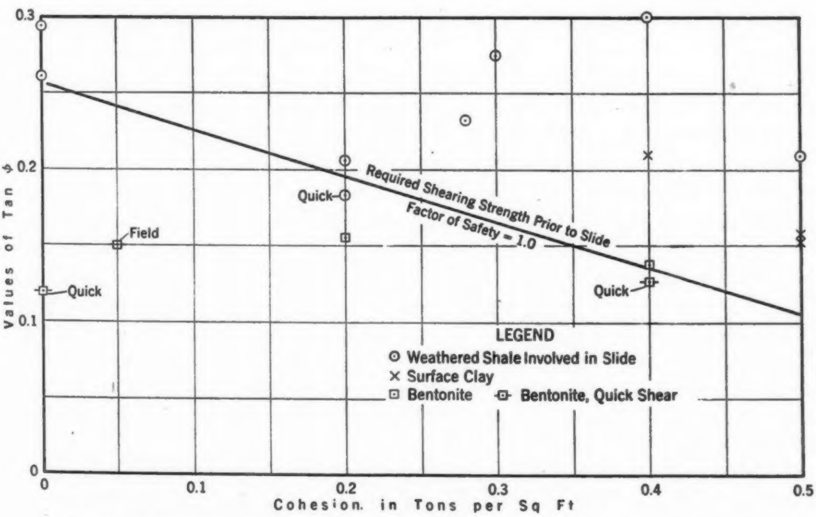


FIG. 10.—VALUES OF COHESION AND FRICTION FOR SOIL ALONG RIGID BOUNDARY FOR A SAFETY FACTOR OF 1.0

A stability analysis was made on the same cross section using the static slide method. Since the weakest plane across the dam passes through very little shell material, the strength of the core is the governing factor. If a value of tan ϕ = 0.3 is used for the core material, this method checks the elastic theory method. On the other hand, if a value of tan ϕ = 0.6, which

is more in line with the test data, is used, the section is shown to be stable by this method. The failure plane as shown in the shale for all stability analyses was determined from the foundation explorations after the slide.

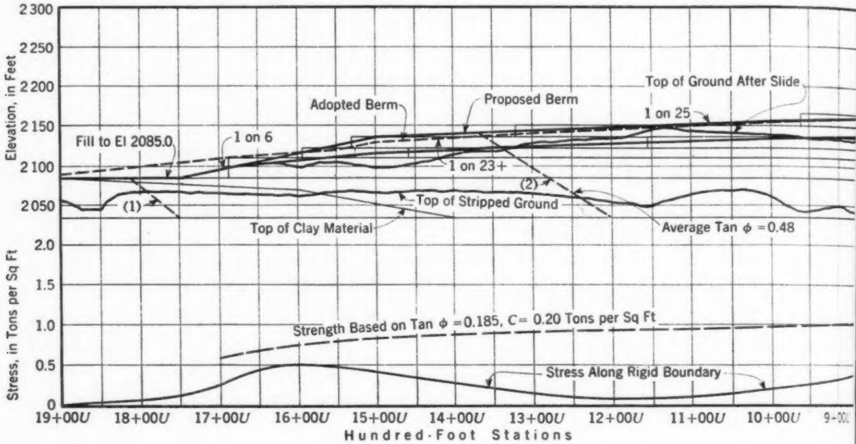


FIG. 11.—STRESS AND STRENGTH ALONG

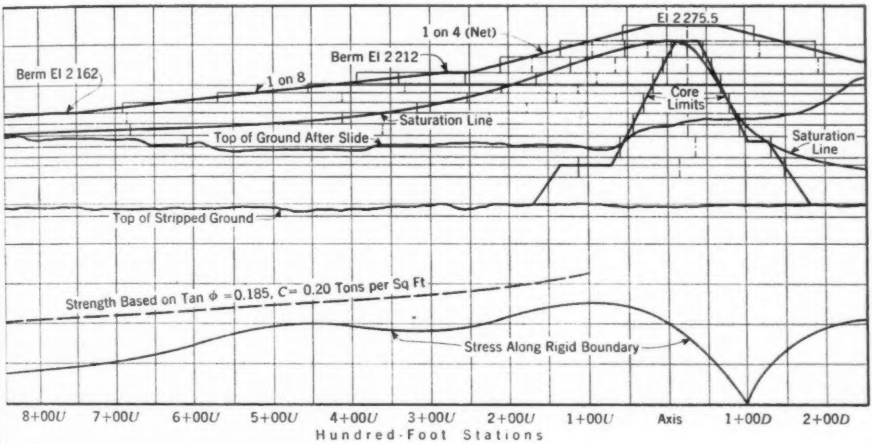
Values of cohesion and friction were determined from consolidated shear tests on undisturbed samples.

The results of these analyses on the sections prior to the slide checked against the test data further show that the failure did occur in the weathered shale and bentonite, and the results also furnish values for the coefficient of friction and cohesion which take into consideration all the variables including the excess hydrostatic pressure. In this way, a reasonable over-all value of the shearing strength of the foundation was determined for use in the re-design.

Reconstruction of the Slide Area.—In all stability analyses for the re-design of the dam in the slide area, values of $\tan \phi = 0.185$ and $C = 0.2$ tons per sq ft were used as determined from the analyses of the dam section prior to the slide. The re-design was based on two criteria established by the Board of Consultants: (1) That the factor of safety of the re-designed section should be greater than 1.5, using values of $\tan \phi = 0.185$ and $C = 0.2$ tons per sq ft; and (2) that no point in the foundation should be overstressed as determined by the elastic theory method.

Figs. 11 and 12 show the stability analysis by the Fort Peck elastic theory method for the re-designed section at Station 15+00. It will be noted in Fig. 11 that in all cases the shearing stress in the shale is well below the shearing strength at any point, and the safety factor against failure of the main dam is 1.95 (Fig. 12) and against failure of the outer slope of the berm is 1.85. The saturation line shown in Fig. 11 represents the worst condition in the upstream shell during dredging operations. As before, all the fill materials were assumed to be homogeneous and isotropic and the effect of the seepage forces was included in the analysis.

Part of Dam Not Affected by Slide.—In checking the design of the dam that was unaffected by the slide, the following criteria were established: (*a*) The factor of safety should be greater than 1.5, using the minimum shearing



RIGID BOUNDARY, RECONSTRUCTION PLAN

strength as determined from previous investigation; and (*b*) no point in the foundation should be overstressed.

In analyzing the center section of the dam over the plastic clay, the results of "quick" or unconsolidated shear tests were used since they gave minimum values. No credit was taken for the increase in strength due to the consolidation in the clay strata which had occurred during construction. A maximum settlement of the embankment had already exceeded 12 ft. Most of this settlement is due to consolidation of the plastic clay. The minimum safety factor as determined for this section of the dam, therefore, is less than those that will actually exist in the completed structure. Using the minimum shear values, the minimum safety factor was found to be 1.5.

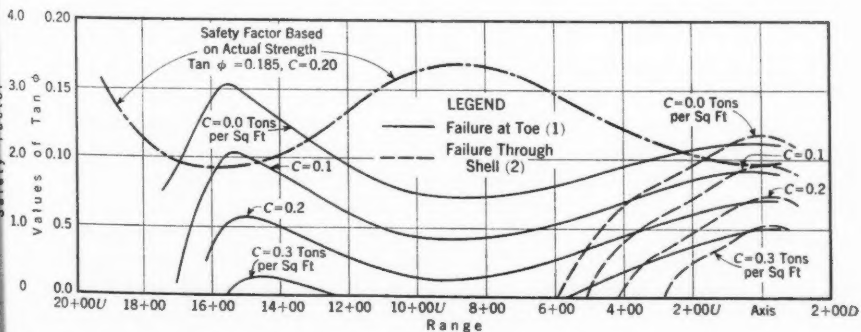


FIG. 12.—RELATION OF COHESION AND TAN ϕ ALONG RIGID BOUNDARY FOR A SAFETY FACTOR OF 1.0

A stability analysis was made for possible failure occurring in the shale on the west abutment at Station 85+00. The minimum safety factor was found to be 1.5. This analysis also has a hidden safety factor. An analysis of this section prior to construction of the berm, assuming that it only had a safety factor of 1.0, gave values of friction and cohesion that checked the value of $\tan \phi = 0.20$ and $C = 0.2$ tons per sq ft used in the re-design. Obviously then, this section had a safety factor greater than one, since it did not fail. Another condition that adds to the safety factor, but which was difficult to take into consideration in the stability analyses, is that the shale drops off rapidly at this abutment which leaves only a short distance along the center line comparable to the section as analyzed.

Taking into consideration all the hidden as well as the apparent safety factors, it is the opinion of the writer that the dam, as re-designed, will have a safety factor, throughout, of approximately 2.0.

Conclusions

A number of reassuring conclusions can be drawn from the investigation of the Fort Peck Slide, with emphasis on two:

(1) It can be concluded definitely that the hydraulic fill was not at fault. It was the general opinion of the investigating engineers and other engineers having an intimate knowledge of the project that the fill material performed excellently even under the most difficult circumstances existing during the slide.

(2) Furthermore, it is encouraging to note that no major changes in soil testing or methods of stability analysis were considered necessary as a result of the slide investigation, although comparison of the "static slide method" with the "elastic theory method" of stability analysis did indicate that the latter was to be preferred for the conditions encountered at Fort Peck.

The paramount question in every one's mind may be stated as follows: How can similar conditions be avoided or anticipated in the future? It is apparent, of course, since the slide did occur, that the foundation explorations and investigations, extensive as they were, did not indicate to the geologist and engineer the extent of the weathering in the shale and bentonite seams underlying this part of the dam; nor did they indicate that high hydrostatic pressures would develop in the bentonite seam due to the superimposed load of the dam. Therefore, the major problem in this connection is to always obtain representative undisturbed samples of this type of rock, sufficiently large to reveal the true character of the rock to geologists and engineers. The slide investigation at Fort Peck has made a major contribution to this end by demonstrating the feasibility of freezing the saturated overburden and taking 36-in. undisturbed cores from the soil and rock formations. The development of hydrostatic pressure in the rock can be determined by the installation of well points or other means of measuring the water pressure. These pressure installations should be made at the beginning of construction and readings taken periodically. Provisions have been made in the reconstruction plans of Fort Peck for observing the hydrostatic pressure that develops in the rock and for relieving it where necessary.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

PLASTIC THEORY OF REINFORCED CONCRETE DESIGN

By Charles S. Whitney,[1] M. Am. Soc. C. E.

Synopsis

It is the purpose of this paper to present a realistic method for the design of reinforced concrete members that should result in more efficient use of the materials. The theory of elasticity has proved satisfactory for the determination of the forces acting on the various sections of continuous frames or arches and much attention has been devoted to various methods of its application. The result has been a general extension of its use and a marked improvement in the understanding, by designers, of the action of reinforced concrete structures.

Corresponding progress has not been made in proportioning the members of a structure after the external forces have been determined. The theory of elasticity, as applied to design of sections, is too inflexible and inaccurate to be entirely satisfactory. The writer proposes the adoption of the plastic theory which recognizes the true characteristics of the material determined by research since the standard "straight-line theory" was adopted about a generation ago. The plastic theory takes into account the plasticity of the material and is in itself sufficiently "plastic" to be adjusted empirically to actual conditions. It has already been given recognition in the general adoption of the standard column formula, and the formulas proposed herein by the writer extend the same theory to beams, eccentrically loaded columns, and arch ribs. The equations are much simpler than the standard formulas and agree better with test results. They are based on the cylinder strength of the concrete and the yield strength of the steel without the use of modular ratio n. A number of attempts[2,3,4] have been made since 1936 to eliminate the modular ratio, n, but

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **April 15, 1941.**

[1] Cons. Engr., Milwaukee, Wis.

[2] "Bruchzustand und Sicherheit im Eisenbetonbalken," by Rudolf Saliger, *Beton und Eisen*, October, 1936, p. 317.

[3] "The Modular Ratio—A New Method of Design Omitting m," by K. Hajnal-Konyi (and discussion by various others), *Concrete and Constructional Engineering*, January, February, March, May, June, July, August, September, October, 1937.

[4] "Considérations sur le Calcul et la Sécurité des Pièces Fléchies. Moments de Rupture," by R. Coppée, *Publications*, International Assoc. for Bridge and Structural Eng., Vol. 3, 1935, p. 19.

none of them is as practical or as well adapted to American practice as that given herein. The writer has previously published parts of this theory,[5,6] but has now improved and extended the equations so that he hopes they may be recognized as forming a satisfactory basis for practical design and further research.

INTRODUCTION

There are three general steps in the design of an efficient structure:

(1) The determination of service conditions depending on the purpose and location of the structure, such as the nature and intensity of loading, the foundation conditions, and the exposure to deteriorating influences;

(2) The computation of the external forces to which the various sections of all the members will probably be exposed during the life of the structure; and

(3) The proportioning of the different parts of the structure so that the limits of their useful strength may be reached, but not exceeded, under the loading specified.

Step 1 leads an experienced designer to the selection of materials and a preliminary layout of the structure. It has been demonstrated by many investigations that, in the case of frames or arches of reinforced concrete, the external forces can be computed satisfactorily (Step 2) by means of the theory of elasticity, based on the assumption of direct proportionality of stress to strain. In spite of the plasticity of concrete this is true within the limits of the useful strength of the structure, except in some special cases where cracking at certain sections sufficient to cause redistribution of moments might not be detrimental. In such special cases, advantage might be taken of the increased strength due to redistribution of moments, but this must be done with caution.

A number of tests since about 1936[7] have shown that the maximum strength of a redundant structure is not determined by the first section to reach its maximum strength but by the combined strength of the redundant and primary members or sections, as explained in 1939 by J. A. Van den Broek,[8] M. Am. Soc. C. E.

Ordinarily, in reinforced concrete structures, the useful load limit of the structure occurs when the first section reaches its allowable limit, as determined by the permissible cracking or yielding of the concrete while the steel stress is still considerably below its yield point. For this reason, it is best to design the structure so that, as nearly as practicable, all the sections involved may reach their allowable limits without exceeding them under the service loading.

Under such a condition, the theory of elasticity is the best guide to the forces acting on the different sections.[7] The probable redistribution of moments

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[5] "Design of Reinforced Concrete Members under Flexure or Combined Flexure and Direct Compression," by Charles S. Whitney, *Journal*, Am. Concrete Inst., March-April, 1937, p. 483.

[6] "Eccentrically-Loaded Reinforced Concrete Columns," by Charles S. Whitney, *Concrete and Constructional Engineering*, London, November, 1938.

[7] "The Redistribution of Moments in Reinforced Concrete Beams and Frames," by W. H. Glanville and F. G. Thomas, *Journal*, Inst. of Civ. Engrs., June, 1936, p. 291.

[8] "Theory of Limit Design," by J. A. Van den Broek, *Transactions*, Am. Soc. C. E., Vol. 105, 1940, p. 638.

caused by yielding under higher loads than the design loads will effect the factor of safety of the structure against collapse and aid in the selection of the proper allowable limits of the strengths of the different sections.

The third step in the design is the proportioning of the different parts of the structure so that the limits of their useful strength may be reached but not exceeded under the loading specified. In dealing with a plastic material such as concrete, this limit can best be determined by a theory that recognizes its true characteristics. The plastic theory is based as closely as practicable on the actual behavior of concrete in combination with steel and makes possible the determination of the useful strength of concrete members with more accuracy than the present standard straight-line stress theory. The strength and reliability of the present-day concrete, and the detailed knowledge gained by practical research certainly justify a complete re-examination of the methods of design.

The Stress-Strain Characteristics of Concrete

The behavior of concrete under stress is extremely complex because it is influenced by combinations of so many variables that even under well-controlled conditions the prediction of true unit stress is not possible.

The classic conception of the stress-strain curve is that of a parabola with its lower part approximately straight and ending at the top tangent to the horizontal. The lower third of the curve being nearly straight, it formed the basis of the straight-line formula in which working stresses are used. The entire curve was used in the parabolic formula based on ultimate strength. Both of these formulas involve Young's modulus of elasticity of the concrete, which has been proved inaccurate by exhaustive research because the stress-strain relation is an indeterminate variable depending on the concrete, the manner and intensity of loading, and on time. Change in strain with time is due to plastic flow and shrinkage.

After years of extensive researches, R. E. Davis, M. Am. Soc. C. E., H. E. Davis, Assoc. M. Am. Soc. C. E., and Elwood H. Brown state that[9]

> "* * * it is not yet possible quantitatively to state with any degree of certainty what is likely to be the magnitude of either plastic flow or shrinkage under the conditions which surround any given concrete structure; nor is it possible quantitatively to state with accuracy what is the effect of plastic flow and shrinkage upon the magnitude and distribution of stresses."

It has been shown that deformations of stressed concrete change with time, and that stresses in a restrained member are greatly affected by plastic flow.[10] Even short-time measurements are affected by plasticity. On account of the variability of this stress-strain relation, it is not possible to determine the unit stresses at any particular point in the concrete member by measuring strains, except to a very limited degree.

The modulus of elasticity of concrete used in design formulas is an empirical value assumed because it appeared to make them give satisfactory results.

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[9] "Plastic Flow and Volume Changes of Concrete," by R. E. Davis, H. E. Davis, and Elwood H. Brown, *Proceedings*, A. S. T. M., Pt. II, Vol. 37, 1937, p. 318.

[10] "Plain and Reinforced Concrete Arches," by Charles S. Whitney, *Journal*, Am. Concrete Inst., March, 1932.

However, the results cannot be made entirely satisfactory, even by this method of choosing n.

An almost universal misconception of the behavior of concrete in flexure is the assumption that at maximum load the highest unit stress is at the outer surface. The descending branch of the compression stress-strain curve has been generally disregarded because it is difficult to measure and has been considered of no particular interest.

Fig. 1 shows some typical stress-strain curves for concrete cylinders selected from those measured in 1938 by O. G. Kiendl and J. A. Maldari.[11] It will be noted that the maximum stress occurs at a strain of approximately 0.002 in. per in., and that the failure does not occur suddenly but that the stress is reduced gradually as the strain continues to increase.

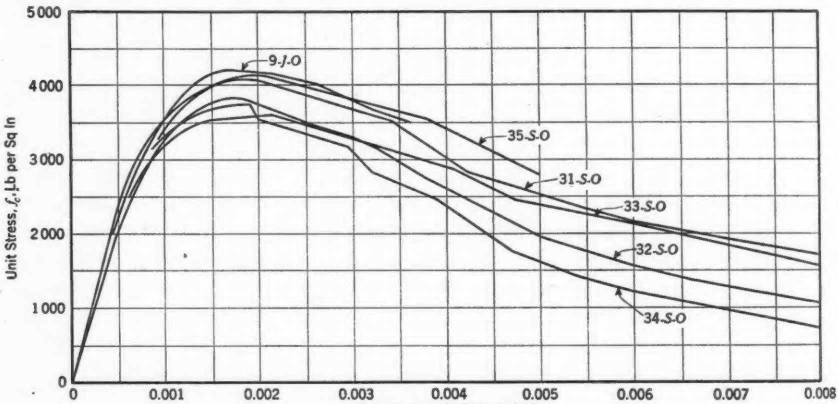

FIG. 1.—TYPICAL STRESS-STRAIN CURVES FOR CONCRETE CYLINDERS IN COMPRESSION

In making these tests, the machine head was moving continuously at the rate of 0.06 in. per min. The effect of time on the stress-strain relation is believed to be similar to that in the case of tests on beams. It is probable, therefore, that these curves indicate the form of compressive stress distribution in a beam at ultimate load in the concrete in the zone where the unit strain exceeds about 0.002 in. per in. In the case of a beam under partial load for a long time, plastic flow will modify the stress-strain relation so that the descending portion of the curve is perhaps of no significance. It is the writer's purpose in this paper to present a method of predicting ultimate strengths. No attempt will be made to determine actual stresses.

In considering the effect of the large strains occurring in a beam as the maximum load is approached it should be remembered that, under favorable circumstances, concrete strained beyond its point of maximum resistance will regain its strength after a rest.[12]

[11] "A Comparison of Physical Properties of Concrete Made of Three Varieties of Coarse Aggregate," a thesis by Oscar G. Kiendl and Joe A. Maldari, presented to the University of Wisconsin, Madison, Wis., in 1938, in partial fulfilment of the requirement for the degree of Bachelor of Science in Civil Engineering.

[12] "Autogenous Healing of Concrete," by M. W. Loving, Assoc. M. Am. Soc. C. E., *Bulletin No. 13*, Am. Concrete Pipe Assoc., 1936; and "Autogenous Healing of Cement and Concrete," by Leslie Turner, London, International Assoc. for Testing Materials, April, 1937.

Because the straight-line formula applied to test results indicates that the concrete stress at maximum load in an over-reinforced beam is much higher (about 50%) than the cylinder strength, and because the maximum strain in the beam is much higher than that in the cylinder at maximum load, it has been concluded quite generally that the concrete does not behave the same in the beam and in the cylinders. Although there is undoubtedly some difference due to the shape of the unit and the distribution of stress, it will be shown that this difference may not be substantial and that the behavior in the compression side of the beam may be fully explained by the stress-strain characteristics of the cylinder.

It is obvious from a comparison of the different curves of Fig. 1 that even in controlled tests there are such great differences in the strains that no very refined theory can be justified. There are general characteristics, however, that form the basis of the plastic theory and indicate the form of the equations, leaving certain empirical constants to be determined by actual tests on beams. In this manner, simple formulas can be derived that have a more satisfactory theoretical basis than the present standard formulas and that give more accurate results.

The ideal stress-strain curve in Fig. 1 was derived from cylinder tests of concrete with a strength of about 4,000 lb per sq in. It has been proved that the strain in the compression side of a beam increases practically in direct proportion to the distance from the neutral axis. Therefore, the ideal curve to the left of any particular ordinate corresponding to the strain at the outer surface of the beam will represent the variation of unit compression stress in the concrete. If the strain in the steel is known, the location of the neutral axis and the resisting moment of the beam can be computed. The tension in the concrete will be neglected because the effect is small at high loads and the error is on the side of safety.

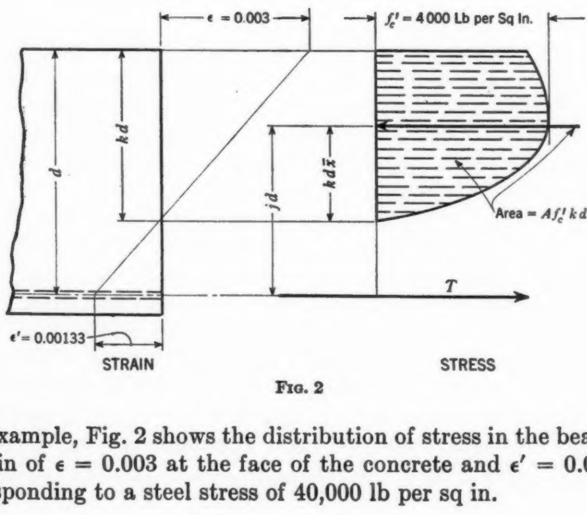

Fig. 2

As an example, Fig. 2 shows the distribution of stress in the beam assuming a unit strain of $\epsilon = 0.003$ at the face of the concrete and $\epsilon' = 0.00133$ at the steel corresponding to a steel stress of 40,000 lb per sq in.

The computed value of k is $\frac{\epsilon}{\epsilon + \epsilon'} = 0.692$. By integrating the stress-strain curve from zero to the ordinate where the strain ϵ is 0.003, it is found that the area is $0.803\ \epsilon f_c'$ in which f_c' is the maximum concrete stress and ϵ is the maximum strain. Furthermore, the center of gravity of the area is $\bar{x}\ \epsilon = 0.571\ \epsilon$ from the origin. Applying this area to the depth of the beam above the neutral axis, substituting $k\ d$ for ϵ, the area represents the total compressive force in the beam; and it equals $C = 0.803\ f_c'\ k\ d = 0.556\ f_c'\ d$. Then the moment arm from the steel to the center of compression is $j\ d = d - k\ d\ (1 - 0.571) = 0.703\ d$. The bending moment for a unit width of beam is then:

$$M = 0.556\ f_c'\ d \times 0.703\ d = 0.391\ f_c'\ d^2 \quad \ldots\ldots\ldots\ldots (1)$$

Corresponding values of moment have been computed for other concrete and steel strains (see Fig. 3(*a*)), using the formula

$$\frac{M}{f_c'\ b\ d^2} = k\ A\ (1 - k + k\ \bar{x}) \quad \ldots\ldots\ldots\ldots (2)$$

in which

$$A = \frac{\int f_c\ d\epsilon}{f_c'\ \epsilon} \quad \ldots\ldots\ldots\ldots (3a)$$

and

$$\bar{x} = \frac{\int \epsilon\ dA}{A\ f_c'\ \epsilon^2} \quad \ldots\ldots\ldots\ldots (3b)$$

In these computations, the values of distance $\bar{x}$ and area A given in Fig. 3(*b*) were used, based on the ideal stress-strain curve. The maximum ordinates in Fig. 3(*a*) give the maximum resisting moments of the beam. They are significant quantitatively only in a general way but they are of great value in indicating what is happening in the beam. The actual maximum value can best be determined from tests on beams.

It is particularly interesting to note that the maximum bending moment is not developed when the stress at the surface of the beam reaches the maximum cylinder strength. As the bending increases, the surface stress decreases but the resisting moment of the beam continues to increase until the strain at the surface is roughly twice that of the cylinder at maximum load. This ratio will vary with the quality of the concrete and the steel strain.

These large strains in the concrete are principally plastic and result in a redistribution of stress as the moment is applied. This yielding brings the center of the compressive stress much nearer the tensile steel than indicated by the standard straight-line formula. A very important feature of this redistribution of stress is that it apparently does not affect the fatigue limit of the beam adversely. Tests made by J. L. Van Ornum,[13] M. Am. Soc. C. E., on heavily reinforced beams that failed in compression showed that the fatigue limit at which beams failed under repeated loading was between 50% and 60% of the static strength and practically the same percentage as the fatigue limit of

[13] "The Fatigue of Concrete," by J. L. Van Ornum, *Transactions*, Am. Soc. C. E., Vol. LVIII, June, 1907, p. 294.

cylinders under repeated compression. The strength of the beams under 1,000 repetitions was about 70% of the static strength.

Referring to the ideal curve in Fig. 3, it will be seen that a moment of 50% of the ultimate may produce initially a stress greater than 80% of the cylinder strength. It might be expected that a repetition of such load would cause failure, but the plasticity of the concrete at such a high stress causes a readjust-

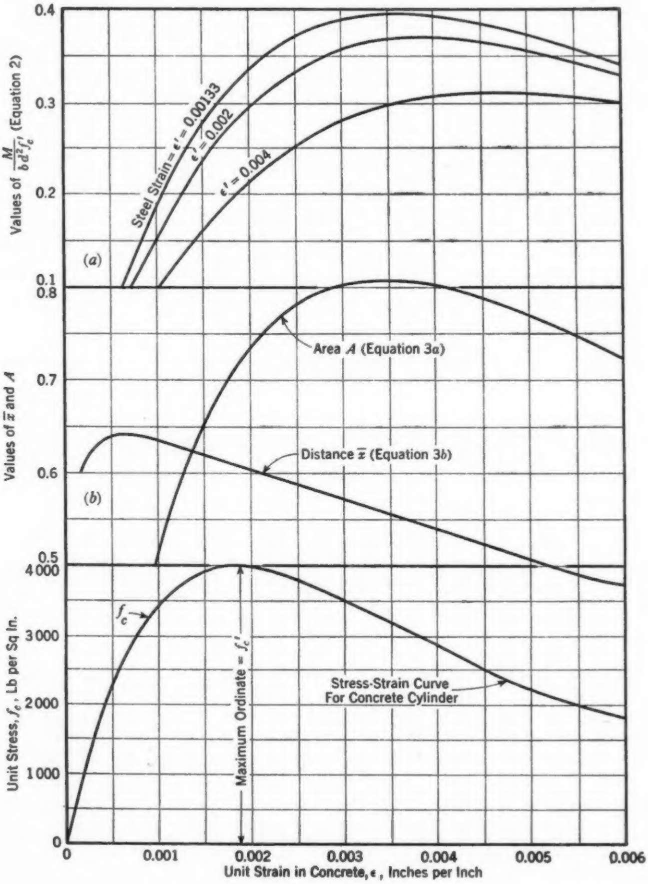

Fig. 3

ment before failure takes place. This is important because it indicates that although the stress in the outer portion of the beam may be higher than the fatigue limit as shown by cylinders, there is a redistribution of stress in the beam so that the fatigue limit of the beam is more than 50% of the ultimate static strength of the beam.

It has been proved that plastic flow is approximately proportional to stress at low stresses, but for higher stresses the plastic strain increases much more

rapidly than the stress.[14] This is to be expected from the shape of the ordinary stress-strain curve.

It is obvious that if the fatigue limit is a reasonably definite percentage of the ultimate strength, the ultimate strength is itself a better measure of the useful strength of the beam than is the unit stress calculated by the standard straight-line formula based on an arbitrary value of the modulus of elasticity. The useful strength can then be determined most effectively by applying a factor of safety to the ultimate strength.

Professor Van Ornum also found that the deflections of beams under repeated loading below the fatigue limit became elastic after a number of repetitions, and the modulus of elasticity was reduced to about 80% of the initial value.

No information is at hand regarding the effect of full reversal of loading on heavily reinforced sections, but this is not a usual condition in concrete structures. Beams with about 0.3% of steel showed about the same fatigue limit under reversed loading as under repeated. Maximum loading conditions occur so infrequently in the life of most structures that this may not be important ordinarily; but more information should be obtained.

Since the stress-strain curves are so variable and indeterminate (as indicated by Fig. 1), a very exact formula based on this relation is not practical. Fortunately it is practicable to derive a simple formula that will give the approximate location of the center of the compression force more satisfactorily than the present standard formula. A simple expression for the maximum limiting value of the compression force and the bending moment can also be determined from beam tests.

Rectangular Beams and Slabs

If it is assumed that the stress distribution in a concrete beam at failure has the shape of the cylinder stress-strain curve shown in Fig. 4, the total com-

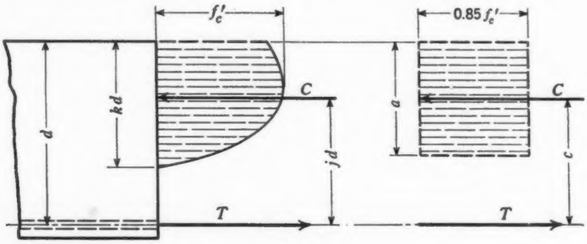

Fig. 4

pression, C, is the area bounded by the curve. The line of action of C lies through the center of gravity of this area. From an examination of the ideal stress-strain curve in Fig. 3, it is found that, if the actual area is replaced by an equivalent rectangular area of width equal to $0.85 f_c'$ and depth equal to a (as shown in Fig. 4), the location of the center of gravity of this rectangle corresponds closely with that of the actual area.

[14] "Short-Time Creep Tests of Concrete in Compression," by R. S. Jensen and F. E. Richart, M. Am. Soc. C. E., *Proceedings*, A. S. T. M., Pt. II, Vol. 38, 1938, p. 417.

In other words, the actual irregular stress block is replaced for simplicity with a rectangular stress block of equal size that has an average stress intensity equal to $0.85 f_c'$. This does not mean that this method is based on the assumption of rectangular stress distribution. The use of the rectangle is merely a mathematical device to approximate the effect of the true distribution. It would be possible to use any curved shape that would give the same area and center of gravity, but the rectangular area appears to be entirely satisfactory and gives the simplest possible mathematical solution. This conception forms the basis of all the following formulas, and they give good agreement with tests. The assumption of a concrete stress equal to $0.85 f_c'$ corresponds to the actual strength in columns and permits a consistent treatment through the full range from pure flexure (eccentricity $e = \infty$) to axial load ($e = 0$).

It should be noted that the depth a does not correspond with the depth to the neutral axis, $k\,d$, and it bears no definite relation to it. The value of $k\,d$ is determined by the strains at the top and bottom of the beam but a is determined by the strengths of the materials.

For practical purposes, the action of the beam may now be described as follows (see Fig. 5):

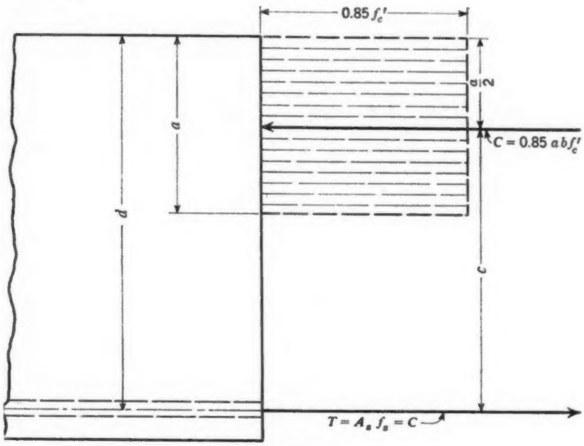

FIG. 5

If the beam is under-reinforced so that the primary failure will occur in the tensile steel, the concrete will crack as the steel stretches, and the equivalent depth of the beam in compression, a, will decrease until the average effective concrete stress reaches the maximum. The concrete will then fail progressively, reducing the lever arm of the steel and causing failure of the beam.

Therefore:

$$a = \frac{A_s f_s}{0.85\, b\, f_c'} = \frac{A_s\, m}{b} \quad \text{(4a)}$$

or

$$\frac{a}{d} = p\, m \quad \text{(4b*)}$$

* Equation Nos. marked thus are essential design formulas.

in which: A_s = area of tensile steel; f_s = yield point stress of steel (or in the case of steel without a definite yield point, the stress producing a total unit strain equal to 0.004 in. per in.); b = width of beam; f_c' = standard concrete cylinder strength; the steel percentage is

$$p = \frac{A_s}{b\,d} \quad \text{(5a)}$$

and

$$m = \frac{f_s}{0.85\,f_c'} \quad \text{(5b)}$$

The lever arm of the steel reinforcement is then

$$c = d - \frac{a}{2} = d - \frac{A_s\,m}{2\,b} \quad \text{(6a)}$$

or

$$\frac{c}{d} = 1 - \frac{p\,m}{2} \quad \text{(6b*)}$$

The ultimate resisting moment of the beam as controlled by steel failure can be written:

$$M = c\,A_s f_s = A_s f_s \left(d - \frac{A_s\,m}{2\,b}\right) \quad \text{(7a)}$$

or

$$\frac{M}{b\,d^2} = p\,f_s \left(1 - \frac{p\,m}{2}\right) \quad \text{(7b*)}$$

Also, for under-reinforced beams,

$$M = 0.85\,f_c'\,a\,b\left(d - \frac{a}{2}\right) \quad \text{(8)}$$

from which

$$\frac{a}{d} = 1 - \sqrt{1 - \frac{2.35}{f_c'}\frac{M}{b\,d^2}} \quad \text{(9a*)}$$

and

$$\frac{c}{d} = \frac{1}{2}\left(1 + \sqrt{1 - \frac{2.35}{f_c'}\frac{M}{b\,d^2}}\right) \quad \text{(9b*)}$$

The foregoing expressions are extremely simple and are independent of n. It should be noted that all of the equations in this paper apply only to ultimate load conditions and are not intended to predict stresses under working loads.

The required steel area is

$$A_s = \frac{M}{c\,f_s} \quad \text{(10*)}$$

In case the moment is less than that required to cause a primary compression failure in the concrete, from Eq. 7b

$$p = \frac{1}{m} - \sqrt{\frac{1}{m^2} - \frac{2}{f_s\,m}\frac{M}{b\,d^2}} \quad \text{(11*)}$$

The limiting value of the depth of equivalent compression, a, for equal concrete and steel strengths in flexure can best be determined experimentally. If there is at least sufficient steel to fully develop the strength of the concrete, additional steel does not materially increase the strength of the beam.

The limiting value of $\frac{a}{d}$ computed from tests reported by Inge Lyse, and the late W. A. Slater,[15] Members, Am. Soc. C. E., is given in Table 1. Eliminating

TABLE 1.—BEAM TESTS REPORTED BY SLATER AND LYSE

Group No.	d (in.)	b (in.)	Ratio p	Cylinder strength (lb per sq in.)	Actual maximum moment (lb-in.)	Eq. 9a $\frac{a}{d}$	Eq. 9b $\frac{c}{d}$	Computed maximum moment from Eq. 12 (lb-in.)	Actual Maximum Moment Divided by Maximum Moment as Computed by		
									Eq. 12	A.C.I. formula	Parabolic formula
1	10.2	8.2	0.021	1,390	597,000	>1.0[a]	....	396,000	1.508[a]	2.06[a]	1.50[a]
2	10.3	8.2	0.028	2,790	818,000	0.544	0.728	811,000	1.007	1.51	1.03
3	10.3	8.2	0.037	4,070	1,152,000	0.515	0.742	1,183,000	0.973	1.50	0.98
4	10.1	8.2	0.047	4,800	1,360,000	0.554	0.723	1,342,000	1.013	1.54	1.00
5	10.2	8.3	0.056	5,740	1,584,000	0.481	0.759	1,660,000	0.955	1.46	0.95
6	14.2	8.2	0.030	2,590	1,805,000	0.910[a]	0.545[a]	1,430,000	1.262[a]	1.85[a]	1.27[a]
6A	14.1	8.2	0.039	4,130	2,196,000	0.518	0.741	2,250,000	0.976	1.50	0.97
7	12.2	8.3	0.028	2,950	1,240,000	0.553	0.723	1,220,000	1.016	1.56	1.03
8	8.0	8.1	0.031	2,760	543,000	0.670[a]	0.665[a]	478,000	1.135[a]	1.66[a]	1.13[a]
9	5.9	7.9	0.032	2,900	310,000	0.708[a]	0.646[a]	266,000	1.166[a]	1.69[a]	1.14[a]
10	4.1	8.0	0.030	2,820	129,000	0.552	0.724	127,000	1.016	1.48	1.00
10A	4.1	8.0	0.040	3,810	179,000	0.576	0.712	171,000	1.046	1.55	1.03
Average						0.537	0.732	....	1.00	1.52	1.00
Standard Deviation						±0.024	±0.012	....	±0.024	±0.028	±0.024

[a] These values not included in average.

four groups of beams that appear erratic, the mean value of $\frac{a}{d}$ is 0.537 and $\frac{c}{d}$ is 0.732. The corresponding ultimate resisting moment is given by the expression

$$\frac{M}{b\,d^2} = 0.85\,f_c'\,\frac{a}{d}\left(1 - \frac{a}{2\,d}\right) = \frac{f_c'}{3} \qquad (12^*)$$

For comparison, Table 1 gives the maximum moment for the test beams as computed from the American Concrete Institute (A.C.I.) formula and from the parabolic formula[16] using the measured value of n. The standard deviation is practically the same in the three cases indicating that there is no advantage in using a formula involving n.

Fig. 6 shows the value of $\frac{M}{b\,d^2\,f_c'}$ given by 36 beams tested by Professors Slater and Lyse,[17] and 33 beams tested by the late Richard L. Humphrey, M. Am. Soc. C. E., and Louis H. Losse[18] which had sufficient steel to cause a

[15] "Compressive Strength of Concrete in Flexure," by W. A. Slater and Inge Lyse, *Journal*, Am. Concrete Inst., June, 1930, p. 831.

[16] *Loc. cit.*, p. 861.

[17] *Journal*, Am. Concrete Inst., 1930, p. 850.

[18] "The Strength of Reinforced Concrete Beams," by Richard L. Humphrey and Louis H. Losse, *Technological Paper No. 2*, National Bureau of Standards, 1912.

primary compression failure. The Humphrey beams, with stronger concrete, were not included because there was not sufficient reinforcement to develop such high concrete strength fully although they may have been reported as compression failures.

Confusion no doubt exists because it is difficult to differentiate between primary steel or concrete failures unless the steel percentage is considerably less than the critical required to develop full concrete strength. The difference cannot be detected by measuring the maximum strain in the concrete as it may be as large in either case. A slight stretching of the steel will reduce the concrete compression area and will cause, in an under-reinforced beam, what appears like a concrete compression failure.

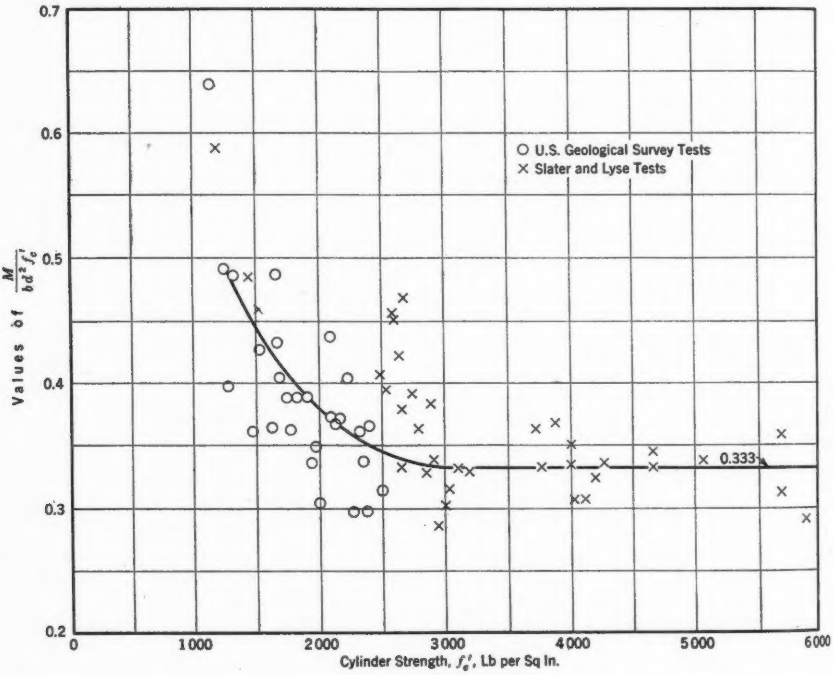

FIG. 6.—VALUE OF $\frac{M}{b\, d^2 f_c'}$ FROM TESTS OF BEAMS

The value of $\frac{M}{b\, d^2 f_c'}$ shown in Fig. 6 is unaffected by the concrete strength between 3,000 and 6,000 lb per sq in., although it rises about 50% for weaker concretes from 3,000 to 1,000 lb per sq in. With present methods and materials it is not probable that structural concrete will have a strength less than 2,500 or 3,000 lb per sq in., and if a weaker concrete is used, a greater factor of safety is appropriate. Therefore, it appears that a value of 0.333 for $\frac{M}{b\, d^2 f_c'}$ can be used generally.

The critical percentage of steel required to develop the full compressive strength of the concrete is (see Eq. 4):

$$p_0 = \frac{0.537}{m} = 0.456 \frac{f_c'}{f_s} \qquad (13^*)$$

For beams with less steel, the allowable bending moment is given by Eq. 7. Table 2 shows the results of the application of Eq. 7 to 72 beams of the series

TABLE 2.—COMPARISON OF COMPUTED AND ACTUAL ULTIMATE STRENGTH OF BEAMS

Item No.[a]	Concrete: Age, in weeks	Concrete: Cylinder strength (lb per sq in.)	Steel: d (in.)	Steel: 100 p	Steel: Yield point (lb per sq in.)	Actual maximum moment (lb-in.)	$\frac{c}{d}$ from Eq. 9b	Computed maximum moment from Eq. 7 (lb-in.)	Actual maximum moment divided by computed
				(a) GRANITE AGGREGATE					
1	4	3,241	10	0.49	42,490	177,000	0.9621	160,500	1.103
2	4	2,807	10	0.98	43,043	321,000	0.9115	307,500	1.043
3	4	3,158	9½	1.96	41,857	492,000	0.8350	507,000	0.970
4	52	5,589	10	0.49	42,573	193,600	0.9780	163,500	1.183
5	52	5,086	10	0.98	42,320	340,000	0.952	319,000	1.066
6	52	4,957	9½	1.96	41,877	543,000	0.895	545,000	0.996
				(b) LIMESTONE AGGREGATE					
7	4	2,087	10	0.49	41,653	164,000	0.9425	154,000	1.064
8	4	2,600	10	0.98	40,797	291,000	0.9095	289,000	1.006
9	4	2,725	9½	1.96	38,037	471,000	0.815	451,000	1.044
10	52	4,069	10	0.49	41,003	183,000	0.9708	157,000	1.165
11	52	4,360	10	0.98	41,413	317,000	0.9451	307,000	1.033
12	52	4,230	9½	1.96	37,673	532,000	0.890	487,000	1.092
				(c) GRAVEL AGGREGATE					
13	4	3,441	10	0.49	43,447	169,000	0.9635	164,000	1.030
14	4	3,440	10	0.98	42,600	321,000	0.9285	310,000	1.035
15	4	3,376	9½	1.96	38,633	504,000	0.857	482,000	1.045
16	52	5,186	10	0.49	44,140	173,000	0.9754	169,000	1.022
17	52	5,242	10	0.98	40,107	356,000	0.9559	300,000	1.186
18	52	5,467	9½	1.96	38,643	533,000	0.9120	512,000	1.042
				(d) CINDERS AGGREGATE					
19	4	1,629	10	0.49	37,357	163,200	0.9337	138,000	1.182
20	4	1,555	10	0.98	41,170	281,000	0.8475	273,000	1.030
21	4	1,643	9½	1.96	37,887	403,000	0.7315	376,000[b]	1.071
22	52	2,944	10	0.49	38,343	172,600	0.9624	145,000	1.188
23	52	2,619	10	0.98	41,103	306,000	0.9090	293,000	1.044
24	52	2,763	9½	1.96	38,047	488,000	0.827	458,000	1.064
						Average			1.071
						Standard deviation			±0.05

[a] Three beams each item.
[b] Determined by concrete strength, Eq. 12.

tested by Messrs. Humphrey and Losse[18] except Item 21 for which Eq. 12 controls because of low-concrete strength. The average ratio of actual maximum moment to that given by Eq. 7 is 1.071 to 1. Eliminating the low-strength cinder concrete beams (Table 2(*d*)), the ratio is 1.062 to 1. Classified according to age, the ratio is 1.052 to 1 at 4 weeks, and 1.09 to 1 at 52 weeks.

According to percentage of steel, it is 1.117 for $p = 0.0049$, 1.054 for $p = 0.0098$, and 1.042 for $p = 0.0196$, showing the increasing influence of the tension in the concrete as the percentage of steel is reduced. There appears to be no correlation between the modulus of elasticity of the concrete and the accuracy of Eq. 7.

The value of the steel lever arm, $\frac{c}{d}$, from Eq. 6 shows a substantial improvement over the j-value given by the standard formula. This is clearly shown by the aforementioned tests.[17, 18] The general trend of the comparative values is shown in Table 3 computed for different percentages of steel in a

TABLE 3.—COMPARISON OF j AND $\frac{c}{d}$

($f_c' = 3{,}000$ Lb per Sq In., $f_s = 50{,}000$ Lb per Sq In., $n = 10$, and $m = 19.6$)

p	Standard Method		Plastic Theory
	$k = \sqrt{2\,p\,n + (p\,n)^2} - p\,n$	j	$\left(\frac{c}{d} = 1 - \frac{p\,m}{2}\right)$
0.005	0.270	0.910	0.951
0.010	0.3585	0.880	0.902
0.0113	0.376	0.875	0.889
0.020	0.400	0.867	0.804
0.027	0.516	0.828	0.732

beam with 3,000 lb per sq in. concrete and 50,000 lb per sq in. yield point steel. It will be noted that for the smaller percentages of steel, $\frac{c}{d}$ is larger than j and this increase is fully justified by the Humphrey tests. As the percentage of steel increases to more than that commonly used, $\frac{c}{d}$ becomes subtantially smaller than j. The importance of this reduction is shown in the tests by Messrs. Slater and Lyse[15] who reported that the steel stresses at failure of the beams were considerably higher than the theoretical values. Table 4 gives a

TABLE 4.—COMPARISON OF j AND $\frac{c}{d}$ AT FAILURE

Description	Series No. 2	3	4	5
Values of j: Theoretical	0.812	0.801	0.808	0.798
Values of j: Actual	0.718	0.740	0.730	0.735
Plastic theory; values of $\frac{c}{d}$ by Eq. 6	0.728	0.743	0.723	0.759

comparison for four series of beams for which strains were measured close to the ultimate load. The values of j at failure have been estimated from the observed values. It is seen that the actual j was much lower than the theoretical and corresponds very well with $\frac{c}{d}$ as given by Eq. 6.

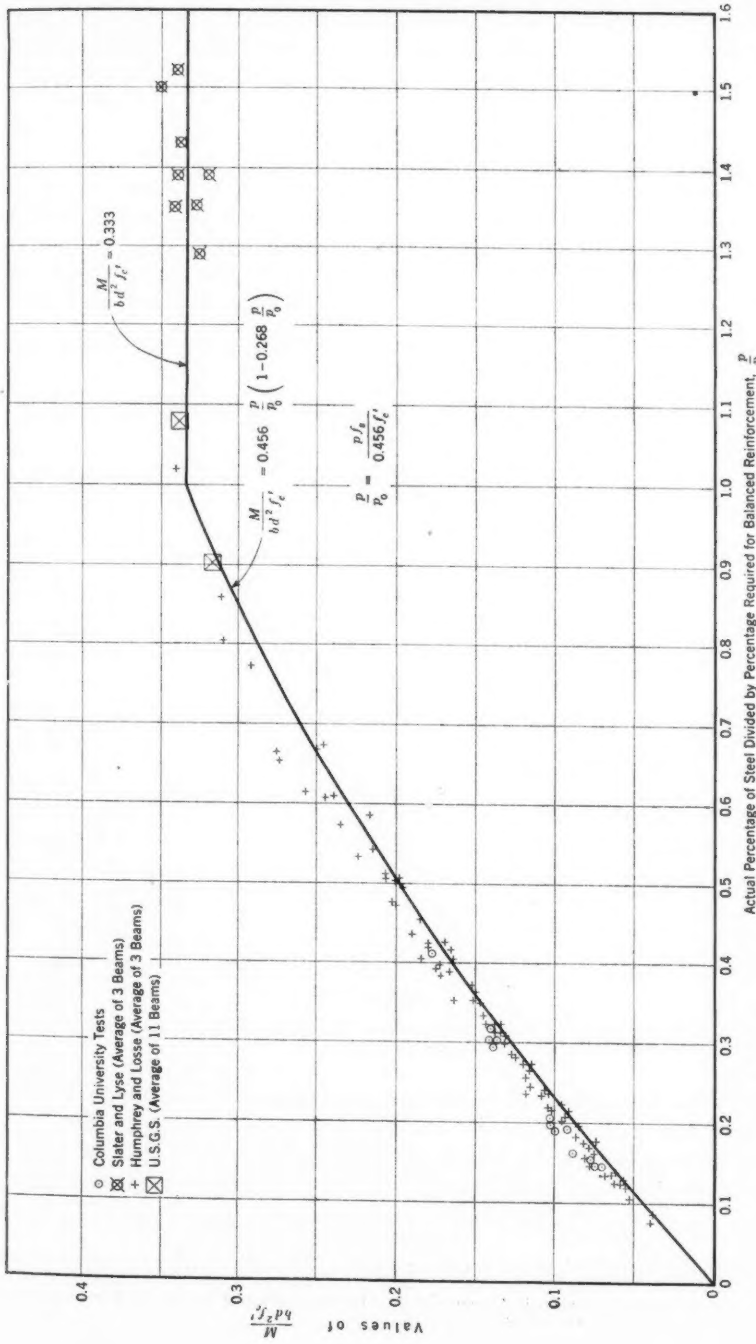

FIG. 7.—COMPUTED ULTIMATE STRENGTH OF CONCRETE BEAMS COMPARED WITH THE RESULTS OF 309 TESTS

A comparison of the results of 309 beam tests with the ultimate strength values given by Eqs. 7 and 12 is shown in Fig. 7 (see Appendix for description). Only well-controlled series of tests have been included and European tests are omitted because of the uncertainty of the translation of cube strength into cylinder strength. It is probable that the cylinder is a more satisfactory index of strength of the concrete in a beam or column than is the cube.

Eq. 7 does not recognize the excess strengths that have been reported in beams and slabs with small percentages of steel,[19] but the use of small percentages of steel is not likely to be as satisfactory in practical construction as it is in the laboratory, and it has not been demonstrated that the excess strength will not disappear under repeated loading. Considerable cracking of slabs in

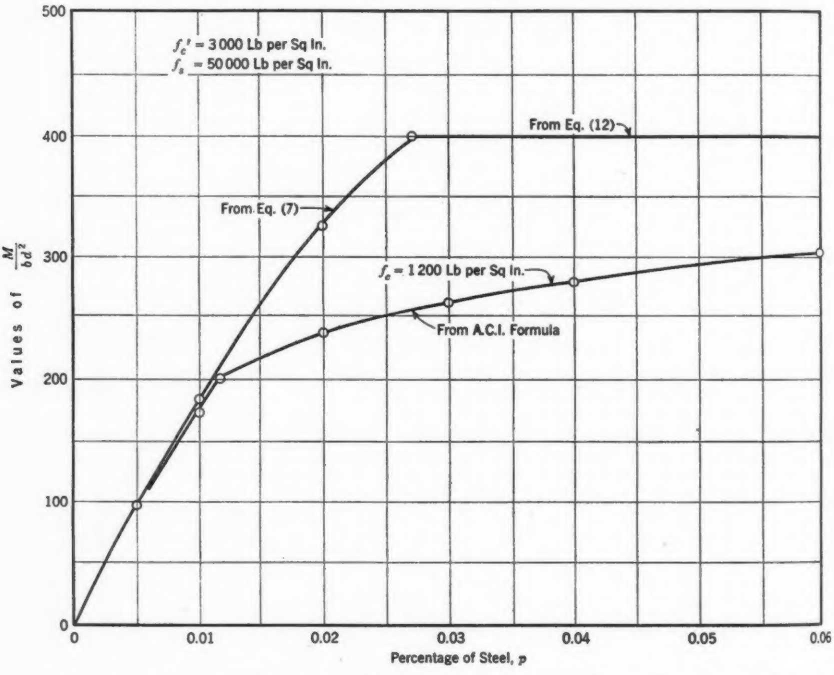

FIG. 8.—COMPARISON OF RESISTING MOMENT OF RECTANGULAR BEAMS COMPUTED FROM VARIOUS FORMULAS

buildings has been observed in the past few years. This is evidently due to shrinkage and lack of plasticity in the relatively high-strength and rapid-hardening concrete, so the use of small percentages of steel should not be encouraged. The effect of the foregoing formulas on rectangular beam design is indicated in Figs. 8 and 9. Fig. 8 shows the allowable $\frac{M}{b\,d^2}$ given by Eqs. 7 and 12 for different values of p with a factor of safety of $2\frac{1}{2}$ as compared with

[19] "A Study of Reinforcement in Concrete Slabs," by Inge Lyse and George R. Wernisch, *Journal*, Am. Concrete Inst., September–October, 1936, p. 1.

the standard formula of the American Concrete Institute when $f_c' = 3{,}000$ lb per sq in. and $f_s = 50{,}000$ lb per sq in. The new formulas raise the critical percentage of steel from 0.0113 to 0.0273, and the allowable $\frac{M}{b\,d^2}$ from 197.5 to 400.

Fig. 9 is a universal design diagram giving values of $\frac{M}{b\,d^2}$ and p for all grades of concrete and steel. The advantage of the simplicity of the basic formulas is

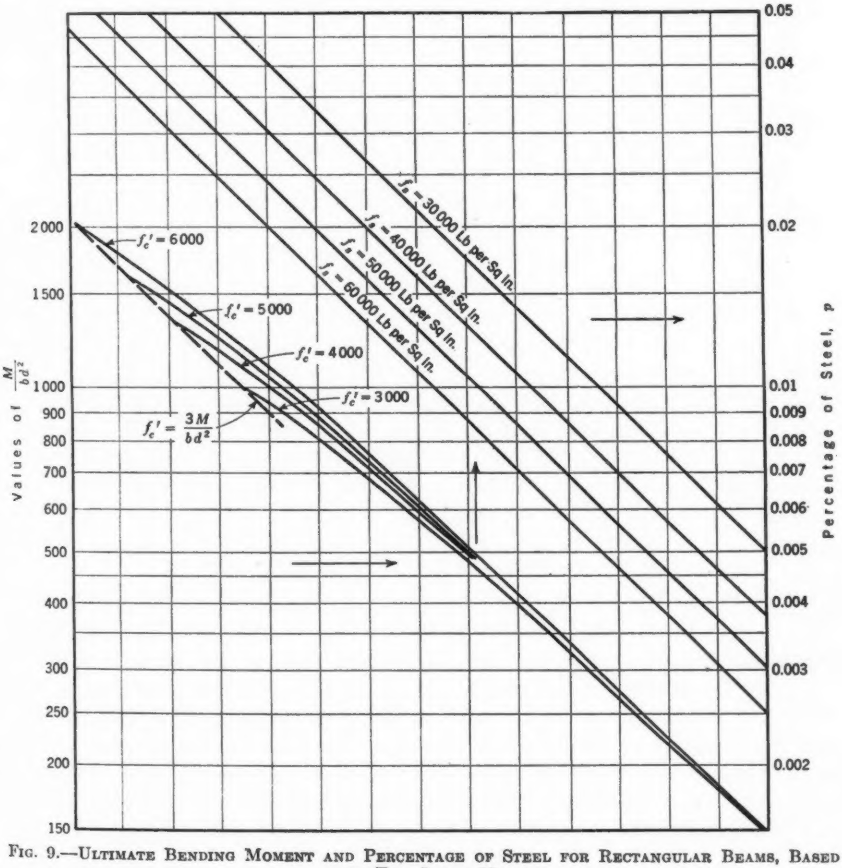

FIG. 9.—ULTIMATE BENDING MOMENT AND PERCENTAGE OF STEEL FOR RECTANGULAR BEAMS, BASED ON EQS. 7 AND 12

obvious, and since the equations conform so well to tests covering the full range of steel and concrete strengths used in practice, there appears to be no reason that they should not be used for design.

DESIGN OF T-BEAMS

The foregoing formulas for rectangular beams (Eqs. 7 and 12) can be directly applied to T-beams when the depth of compression, a, does not exceed the flange

thickness t. For larger moments, when t is less than 0.537 d, it can be assumed that the entire flange area is working at an average stress of $0.85 f_c'$ with the resultant at the center depth of the flange. Then the ultimate moment is given by

$$\frac{M}{b\, d^2} = 0.85\, f_c' \left(1 - \frac{t}{2\, d}\right) \frac{t}{d} \qquad (14a^*)$$

and

$$A_s = \frac{M}{f_s \left(d - \frac{t}{2}\right)} \qquad (14b^*)$$

When the flange thickness is less than 0.537 d and it is desired to take into account the area of the stem between the flange and the line 0.537 d from the top, this may be done by assuming that the area is working at an average stress of $0.85 f_c'$ and taking moments about the center of the steel area.

In applying Eqs. 14 to T-beam design, it is important that special consideration be given to the width of flange that is assumed to be effective. The few tests made by C. Bach[20] to determine the effective width of flange were made on a very low-strength concrete (a cube strength of about 1,700 lb per sq in.) which would probably have a higher relative shearing and tensile strength than the modern high-compressive-strength concrete. For this reason and because of the tendency for shrinkage cracks to form alongside of the beams, even when the slabs are well reinforced, a more conservative value for flange width should be used than that permitted by standard codes. The projection should probably not exceed four times the flange thickness until more definite information is available. With the greater efficiency provided by these formulas this width should be sufficient in most cases.

Beams Reinforced for Compression

Steel in the compressive side of a beam acts as it does in a column. If the beam fails in compression, the concrete strain will be great enough to stress the compression steel to its elastic limit provided it does not slip or buckle.

If the beam fails in the tensile steel, the presence of the compression steel will have comparatively little effect on its ultimate strength. The ultimate bending moment in compression is computed by adding the moment of the steel compressive stress to that of the concrete stress (see Fig. 10). Then the ultimate moment at which the beam will fail in compression, assuming it to be fully reinforced in tension, is

$$M = \tfrac{1}{3}\, b\, d^2 f_c' + d'\, A_s'\, f_s \qquad (15a^*)$$

or

$$\frac{M}{b\, d^2} = \frac{f_c'}{3} + \frac{d'}{d}\, p'\, f_s \qquad (15b^*)$$

in which A_s' is the area of the compression steel, and d' is its lever arm. If the yield point of the compressive steel has a value of f_s' different from f_s, it can

[20] "Versuche mit Eisenbetonbalken," von C. Bach and O. Graf, *Mitteilungen über Forschungsarbeiten auf dem Gebiete des Ingenieurwesens*, Heft 90 and 91, Berlin, 1910.

be taken into account in the equation by changing the value of A_s' used in Eq. 15*a* to the value of $\frac{f_s'}{f_s} A_s'$ without changing the equation. This applies to all equations involving compressive steel.

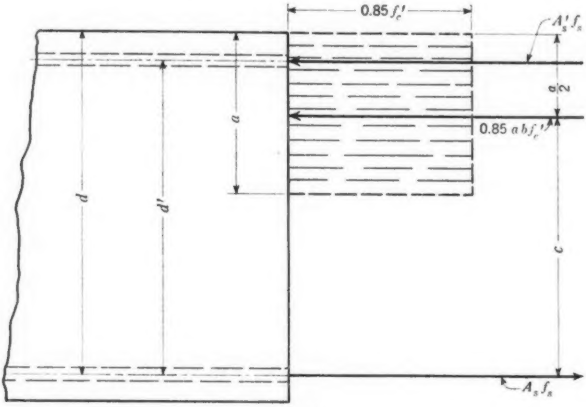

Fig. 10

When the beam is under-reinforced in tension, the lever arm of the tensile steel can be computed as if the compressive steel is stressed to its elastic limit and the remainder of the compression is on the concrete. The calculation is made as if the tensile steel were in two parts, one to balance the compressive steel and the other to balance the stress on the concrete. The ultimate tensile moment is then

$$M = d' A_s' f_s + (A_s - A_s') \left[d - \frac{(A_s - A_s')\, m}{2\, b} \right] f_s \ldots\ldots (16^*)$$

or

$$\frac{M}{b\, d^2} = \frac{d'}{d} p' f_s + (p - p') \left[1 - \frac{(p - p')\, m}{2} \right] f_s \ldots\ldots\ldots (17^*)$$

and the tensile steel required is

$$p = p' + \frac{1}{m} - \sqrt{\frac{1}{m^2} - \frac{2}{f_s\, m} \frac{M - d' A_s'}{b\, d^2}} \ldots\ldots\ldots\ldots (18^*)$$

Eqs. 16 and 17 assume that A_s is greater than A_s'. If A_s' is the greater, the lever arm of the tensile steel can be taken as d' and Eq. 16 becomes:

$$M = d' A_s f_s \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Eqs. 15 to 18 have been checked against the results of tests with satisfactory agreement provided that failure is not caused by shear or bond. The results of tests of beams with compressive steel made by C. Bach and O. Graf at Stuttgart, Germany, and reported by Fritz von Emperger[21] are of interest

[21] "Handbuch für Eisenbetonbau," by Fritz von Emperger, Vol. 1, 3d Ed., p. 225.

because they show the danger of bond failure. The full yield point stress of 35,000 lb per sq in. was developed in compression, but with bars of 60,000 lb per sq in. yield point the failure was in bond before the full strength was developed.

Tests made with twin twisted bars in the compression side of beams[22] indicate that, because of buckling due to the twisting of the bars, they are not as fully effective in compression as they are in tension.

It is extremely important, of course, to provide against failure by bond, shear, and buckling in highly reinforced beams.

Flexure and Direct Load on Rectangular Sections

The strength of the compression side of the section will be the same when the member is subjected to bending and direct load as it is under flexure alone. Therefore, Eq. 15*a* can be used to predict the resistance to compression failure of an eccentrically loaded rectangular section.

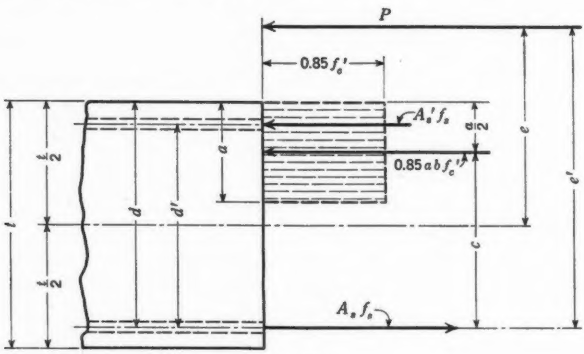

FIG. 11

When there is sufficient tensile steel to prevent a tension failure,[23] the ultimate compressive moment is (see Fig. 11) $M = P\left(e + d - \frac{t}{2}\right) = \frac{1}{3} b\, d^2 f_c' + d' A_s' f_s$ from which, if $d = \frac{t + d'}{2}$,

$$P = \frac{2 A_s' f_s}{\frac{2 e}{d'} + 1} + \frac{b\, t\, f_c'}{\frac{3 t e}{d^2} + \frac{6 d t - 3 t^2}{2 d^2}} \quad \ldots\ldots\ldots\ldots\ldots (20^*)$$

Eq. 20 gives the theoretical value when the eccentricity of the load is greater than the eccentricity of the resisting forces in the compression side of the section. Because of its derivation it has no theoretical meaning for smaller eccentricities but it can be adjusted to the smaller range by making P approach the proper value for an axially loaded column as e approaches zero. The first

[22] "Versuche über Zielsichere Betonbildung und an druckbewerten Balken," by Rudolf Saliger, *Beton und Eisen*, January 20, 1935.

[23] "Plain and Reinforced Concrete Arches," Report of Committee 312, by Charles S. Whitney, Chairman, *Proceedings*, Am. Concrete Inst., September, 1940.

term giving the steel strength needs no adjustment because it equals $2\,A_s'\,f_s$ when e equals zero. It is known that the concrete strength is $0.85\,b\,t\,f_c'$ for axial load so the second term becomes $\dfrac{b\,t\,f_c'}{\dfrac{3\,t\,e}{d^2}+1.178}$, and with this change, Eq. 20 becomes:

$$P = \frac{2\,A_s'\,f_s}{\dfrac{2\,e}{d'}+1} + \frac{b\,t\,f_c'}{\dfrac{3\,t\,e}{d^2}+1.178} \qquad \text{(21*)}$$

which gives the strength of the rectangular member as controlled by compression strength. Eq. 21 gives values which agree satisfactorily with tests, as will be shown subsequently.

The formula for the strength of the member as controlled by ultimate tensile strength of the steel for the larger eccentricities will now be derived. If the member is over-reinforced on the compression side so that the compression steel is sufficient to take the total compression force without help from the concrete, the value of the ultimate load can be expressed by taking moments, and

$$P = A_s\,f_s\,\frac{2\,d'}{2\,e - d'} \qquad \text{(22*)}$$

When there is not sufficient compression steel to take all the compression force, it may be assumed for practical purposes that both the tension and compression steel will be stressed to the yield point at ultimate load, and the remainder of the compression load on the section acts on the compression side of the concrete over an equivalent depth equal to a. This total compression on the concrete is then $0.85\,a\,b\,f_c'$ and it follows that $P + A_s\,f_s = A_s'\,f_s + 0.85\,a\,b\,f_c'$ or, $0.85\,a\,b\,f_c' = P + (A_s - A_s')\,f_s$ and $a = \dfrac{P}{0.85\,b\,f_c'} + (A_s - A_s')\,\dfrac{m}{b}$, in which $m = \dfrac{f_s}{0.85\,f_c'}$.

Then taking moments about the tension steel,

$$M = P\left(e + \frac{d'}{2}\right) = d'A_s'\,f_s + 0.85\,f_c'\,a\,b\left(\frac{d'}{2} + \frac{t}{2} + \frac{a}{2}\right) \qquad \text{(23)}$$

Substituting the previous values and solving for P, we find: $P = 0.85\,t\,b\,f_c' \times$

$$\times\left\{\sqrt{\left[\frac{e}{t} - 0.5 + (p - p')\,m\right]^2 + \frac{2\,d'}{t}\,p'\,m + (p - p')\,m\left[\frac{d'}{t} + 1 - (p - p')\,m\right]} - \left[\frac{e}{t} - 0.5 + (p - p')\,m\right]\right\} \qquad \text{(24a*)}$$

For the symmetrical reinforcement, when $p = p'$, this reduces to

$$P = 0.85\,t\,b\,f_c'\left\{\sqrt{\left(\frac{e}{t} - 0.5\right)^2 + \frac{d'}{t}\,p_t\,m} - \left(\frac{e}{t} - 0.5\right)\right\} \qquad \text{(24b*)}$$

in which, $p_t = p + p' = \frac{A_s + A_s'}{b\,t}$. When there is no compressive steel, $p' = 0$ and the ultimate load equals

$$P = 0.85\,t\,b\,f_c' \left\{ \sqrt{\left(\frac{e}{t} - 0.5 + p\,m\right)^2 + \left(\frac{d'}{t} + 1 - p\,m\right) p\,m} - \left(\frac{e}{t} - 0.5 + p\,m\right) \right\} \ldots\ldots (24c^*)$$

On account of the assumption that the steel on the side of the column away from the load is in tension, Eqs. 24 have no significance when e is very small. That case is covered by Eq. 21.

When the value of $\frac{e}{t}$ is large, it is difficult to obtain satisfactory accuracy in solving Eqs. 24 with a slide-rule. Formulas can be derived in another form to cover the same case in the same manner as Eqs. 16 and 17. This can be done most conveniently in terms of e', the eccentricity measured from the tensile steel.

Assuming the compressive steel to be working at yield-point stress, the percentage of tensile steel required to balance it will be $A_s'\left(1 - \frac{d'}{e'}\right)$. Subtracting this quantity from A_s, the percentage of tensile steel that is dependent on the development of compression in the concrete for its fulcrum to work against is determined. The value of P that can be resisted by these two portions of the tensile steel can be written and combined as follows:

$$P = f_s \left[A_s - A_s'\left(1 - \frac{d'}{e'}\right)\right] \frac{\beta}{e' - \beta} + \frac{d'}{e'} A_s' f_s \ldots\ldots (25^*)$$

in which

$$\beta = \frac{1}{2}\left\{ d + e' - \sqrt{(e' - d)^2 + \frac{2\,e'\,m}{b}\left[A_s - A_s'\left(1 - \frac{d'}{e'}\right)\right]} \right\}$$

When the reinforcement is symmetrical, $A_s' = A_s$ and Eq. 25 reduces to

$$P = \frac{2\,d'\,A_s\,f_s}{e' - d + \sqrt{(e' - d)^2 + \frac{2\,d'\,A_s\,m}{b}}} \ldots\ldots (26a^*)$$

When there is no compression steel $A_s' = 0$ and this becomes

$$P = A_s\,f_s \frac{e' + d - \sqrt{(e' - d)^2 + \frac{2\,e'\,A_s\,m}{b}}}{e' - d + \sqrt{(e' - d)^2 + \frac{2\,e'\,A_s\,m}{b}}} \ldots\ldots (26b)$$

Tables 5 and 6 compare the values of ultimate load given by Eqs. 21 to 26 with

the actual loads found in several series of tests by Messrs. Bach and Graf,[24] F. E. Richart and T. A. Olson,[25] and A. G. Hayden,[26] M. Am. Soc. C. E. In the case of the Bach and Graf tests (Table 5), it was necessary to translate the cube strength into cylinder strength, and it is assumed the strength of the 30 cm cube per sq in. is 1.13 times that of the standard 6-in. by 12-in. cylinder.[27]

TABLE 5.—COMPARISON OF ACTUAL AND COMPUTED ULTIMATE LOADS FOR COLUMNS TESTED UNDER ECCENTRIC LOADS

Eccentricity, e/t	Equation No.	Actual Load (Lb): Ultimate	Actual Load (Lb): First crack	Computed ultimate load (lb)	Col. 3 / Col. 5	Actual Load (Lb): Ultimate	Actual Load (Lb): First crack	Computed ultimate load (lb)	Col. 3 / Col. 5
(1)	(2)	(3)	(4)	(5)	(6)	(3)	(4)	(5)	(6)
		(*a*) SERIES I; $p = 0.005$ AND $p' = 0$				(*b*) SERIES IV; $p = 0$ AND $p' = 0$			
0	21	618,000	....	596,000	1.04	609,000	....	597,000	1.02
0.25	24*c*	....	....	....	....	300,000	208,000	299,000	1.00
0.375	24*c*	....	....	....	....	180,500	91,900	149,000	1.21
0.50	24*c*	204,700	75,700	203,000	1.01	52,800	52,500	0	....
0.75	24*c*	132,700	44,800	123,300	1.07	....	....	....	....
1.25	24*c*	66,000	23,500	62,500	1.07	....	....	....	....
		(*c*) SERIES II; $p = 0.005$ AND $p' = 0.005$				(*d*) SERIES III; $p = 0.0095$ AND $p' = 0.0095$			
0	21	735,000	....	729,000	1.01	892,000	....	835,000	1.07
0.25	21	447,000	293,000	423,000	1.05[a]	496,000	358,000	486,000	1.02[a]
0.50	24*b*	273,000	77,800	257,000	1.06	347,000	99,000	343,000	1.01
0.75	24*b*	153,400	45,200	148,000	1.03[a]	231,500	58,900	219,000	1.05[a]
1.25	24*b*	71,300	22,300	68,500	1.04[a]	118,000	27,700	116,400	1.01

[a] Two tests each; all other determinations based on three tests each.

The Bach and Graf tests (Table 5) show very close agreement with the formulas for the full range, with the actual strengths slightly larger in all cases. They were made on 49 columns about 16 in. square with an equivalent cylinder strength of 2,830 lb per sq in. The steel had yield strengths of 53,700 lb per sq in. and 50,900 lb per sq in. with p equal to 0.005 and 0.0095, and p' equal to zero, 0.005, and 0.0095. The tests therefore covered a very significant range. The details of the sections, and the agreement between the actual and theoretical strengths, have been presented elsewhere.[28]

The Richart and Olson tests (Table 6(*b*)) on knee frames[25] show excellent agreement with the exception of No. 9 which had about 3% of tensile steel and failed at about 85% of the load given by Eq. 26*b*. Cracks formed before

[24] "Tests of Reinforced and Unreinforced Concrete Columns under Axial and Eccentric Load," by C. Bach and O. Graf, *Forschungsarbeiten auf dem Gebiete des Ingenieurwesens*, Vols. 166 to 169.

[25] "Rapid and Long Time Tests on Reinforced Concrete Knee Frames," by F. E. Richart and T. A. Olson, *Journal*, Am. Concrete Inst., March–April, 1937, p. 459.

[26] "Tests of Knees for Continuous Frame Concrete Bridges," by Arthur G. Hayden, *Engineering News-Record*, January 18, 1923, Vol. 90, p. 108.

[27] "Effect of Size and Shape of Test Specimen on Compressive Strength of Concrete," by H. F. Gonnerman, M. Am. Soc. C. E., *Bulletin No. 16*, Structural Materials Research Laboratory, Lewis Inst., Chicago Ill., 1925.

[28] *Journal*, Am. Concrete Inst., Vol. 12, No. 1, September, 1940, p. 1 (compare Figs. 5, 6, 7, and 8).

failure parallel to the compression face at the knee, indicating that it may have been caused by high shearing stresses due to the sharp curvature of the line of compression stress around the angle of the knee. The other frames with about 1% of steel failed primarily in tension and indicated no weakness in compression due to the concentration of stress at the knee when the ordinary percentage of reinforcement is used. More information is needed regarding the knee effect with higher steel percentages.

TABLE 6.—Tests of Reinforced Concrete Knee Frames

Frame No.	Steel Area (Sq In.)		f_c' (lb per sq in.)	Dimensions (In.)[a]			Actual ultimate load[b] (lb)	Computed Ultimate Load		$\frac{\text{Col. 7}}{\text{Col. 8}}$
	A_s	A_s'		d	d'	e'		(Lb)	Eq. No.	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(a) Rapid and Long-Time Tests (b = 12 in. and f_s = 48,300 lb per Sq In.)[e]										
1	1.77	0	3,545	14.50	....	62.5	31,900	31,800[c]	26*b*	1.003
2	1.77	0	3,460	13.64	....	54.07	35,700	34,800[c]	26*b*	1.025
3	1.77	0	3,310	12.78	....	45.64	42,200	38,800[c]	26*b*	1.09
4	1.77	0	3,305	13.64	....	54.07	35,000	34,500[c]	26*b*	1.015
5	1.77	0	3,350	13.30	....	45.64	41,650	41,300[c]	26*b*	1.010
6	1.77	0	3,330	14.50	....	62.50	32,450	31,400[c]	26*b*	1.033
7	1.77	0	2,920	13.30	....	45.64	42,500	40,000[c]	26*b*	1.062
8	1.77	1.77	2,985	14.50	13.0	62.5	32,650	32,400[c]	26*a*	1.008
9	1.77	3.56	3,145	14.50	13.0	62.5	34,350	31,700[c]	22	1.082
10	1.77	1.77	3,035	13.30	11.25	45.64	42,500	41,300[c]	26*a*	1.030
11	1.77	3.56	3,055	13.30	11.25	45.64	43,300	39,500[c]	22	1.096
19	4.84	0	4,185	13.75	....	61.75	57,125	67,700[c]	26*b*	0.845[d]
(b) Knees for Continuous-Frame Concrete Bridges (b = 18 in. and f_s = 45,000 lb per Sq In.)[e]										
A	1.25	0	3,640	9.0	....	37.65	19,300	16,000	26*b*	1.206
B	1.18	0	3,360	9.0	....	37.65	16,000	15,100	26*b*	1.060
C	1.18	0	2,790	9.75	....	39.6	23,330	21,900[c]	26*b*	1.064
D	1.18	0	2,640	9.75	....	39.6	25,600	21,800[c]	26*b*	1.174
E	1.18	0	3,390	9.75	....	39.6	25,800	22,300[c]	26*b*	1.158
F	1.18	0	1,810	9.0	....	37.65	15,900	13,700	26*b*	1.15

[a] In Col. 6, e' is measured perpendicular to the axis of the leg of the member. [b] Average of two frames. [c] $P\sqrt{2}$. [d] Possible shear failure. [e] Part (*a*) refers to footnote 25; and part (*b*) refers to footnote 26.

Eqs. 21 to 26 give the ultimate strengths of rectangular members for the full range of conditions from axially loaded columns to beams under flexure alone. Since it is customary to provide a greater factor of safety for axially loaded columns than for beams, it is necessary to modify some of these formulas for use in design. Without changing their form they can easily be adjusted to any desired combination of beam and column factors of safety.

For instance, in the case of the compression control Eq. 21, it would be reasonable to assume that the factor of safety should vary from that now provided by the present standard formula for axially loaded columns to $2\frac{1}{2}$ for the case of pure bending. When plotted with P as ordinate and $P\,e$, as abscissa, Eq. 21 is a straight line, and it can be adjusted by lowering the y-axis intercept to any desired point.

The standard formula for the axially loaded column is

$$P = 0.4\, A_{st} f_s + 0.225\, A_c f_c' \qquad (27)$$

in which A_{st} is the total area of longitudinal steel; and A_c = area of concrete in compression. Eq. 21 can be adjusted so that P will be $2\frac{1}{2}$ times this value when e is zero and it then becomes

$$P = \frac{2\, A_s' f_s}{\frac{2\, e}{d'} + 1} + \frac{A_c f_c'}{\frac{3\, t\, e}{d^2} + 1.775} \qquad (28^*)$$

Eq. 28 can be used as an ultimate strength formula and will provide the desired variation in factor of safety. The relation between Eqs. 21 and 28 is

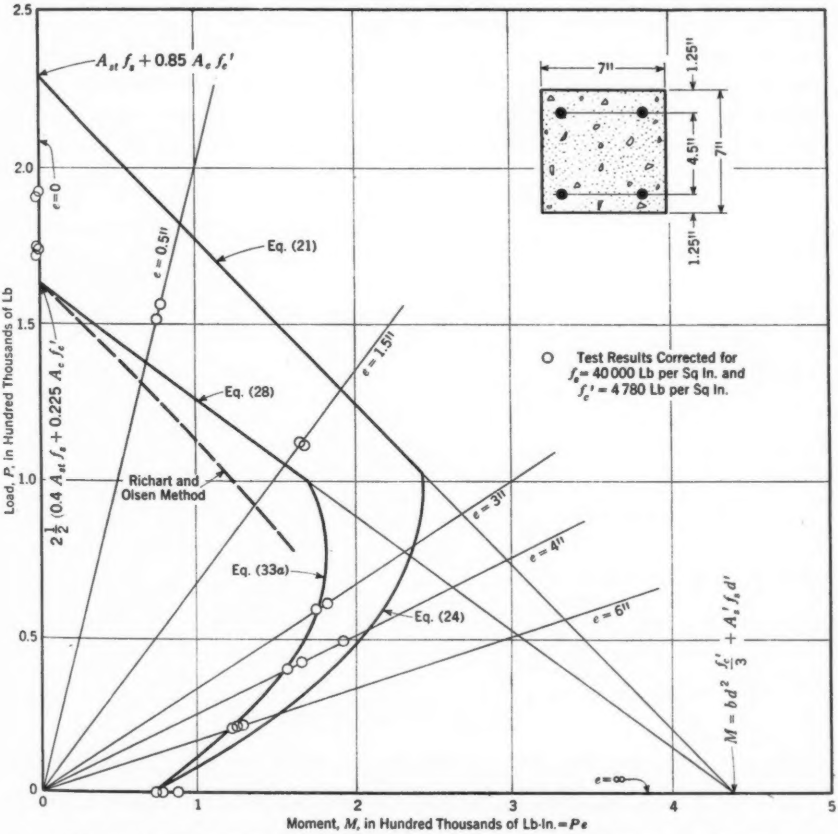

FIG. 12.—TEST RESULTS, SERIES F E, CORRECTED FOR f_s = 40,000 LB PER SQ IN. AND f_c' = 4,780 LB PER SQ IN.

shown by Fig. 12, assuming: d = 5.15 in.; d' = 4.5 in.; $A_s = A_s'$ = 0.392 sq in.; f_s = 40,000 lb per sq in.; f_{cu} = 6,250 lb per sq in.; and f_c' = 4,780 lb per sq in.

The allowable axial load for rectangular tied columns, according to standard practice, is

$$P = 0.8\,(0.4\,A_{st}\,f_s + 0.225\,A_c\,f_c') \qquad (29)$$

and the allowable eccentric load when the reinforcement is symmetrical may be obtained by multiplying Eq. 28 by $\frac{0.8}{2.5}$ which gives

$$P = \frac{0.32\,A_{st}\,f_s}{\frac{2\,e}{d'} + 1} + \frac{0.18\,A_c\,f_c'}{\frac{1.69\,t\,e}{d^2} + 1} \qquad (30^*)$$

Eq. 30 covers the case of the rectangular tied column when compression controls. When the eccentricity is so large that the tension in the steel controls the load, Eqs. 24 can be modified to provide an additional factor of safety for the smaller eccentricities by adding a constant to the eccentricity. Because of the large strains at ultimate load, the deflection of the column (assuming the length limited to ten times t) might possibly increase the eccentricity by as much as $0.1\,t$, and if this amount is added to e used in the derivation of Eq. 24*a*,

$$M = P\left(e + \frac{t}{10} + \frac{d'}{2}\right) \qquad (31)$$

and it becomes: $P = 0.85\,t\,b\,f_c' \times$

$$\times\left\{\sqrt{\left[\frac{e}{t} - 0.4 + (p - p')\,m\right]^2 + \frac{2\,d'}{t}\,p'\,m + (p - p')\,m\left[\frac{d'}{t} + 1 - (p - p')\,m\right]} - \left[\frac{e}{t} - 0.4 + (p - p')\,m\right]\right\} \qquad (32^*)$$

Then Eq. 24*b* becomes, when $p' = p$—

$$P = 0.85\,t\,b\,f_c'\left\{\sqrt{\left(\frac{e}{t} - 0.4\right)^2 + \frac{d'}{t}\,p_t\,m} - \left(\frac{e}{t} - 0.4\right)\right\} \qquad (33a^*)$$

When $p' = 0$, Eq. 24*c* becomes

$$P = 0.85\,t\,b\,f_c'\left\{\sqrt{\left(\frac{e}{t} - 0.4 + p\,m\right)^2 + \left(\frac{d'}{t} + 1 - p\,m\right)p\,m} - \left(\frac{e}{t} - 0.4 + p\,m\right)\right\} \qquad (33b^*)$$

The comparison of Eqs. 33*a* and 24*b* are shown by Fig. 12. The addition of $\frac{t}{10}$ to e is arbitrary and it can be increased or decreased if desired but that amount appears to give good results as compared with tests. Eqs. 25 and 26 are only used for large eccentricities and need no reduction.

The writer has compared[6] the ultimate loads computed from Eqs. 28 and 33*a* with the actual loads found in tests made by Messrs. Richart and Olson[29]

[29] "The Resistance of Reinforced Concrete Columns to Eccentric Loads," by F. E. Richart and T. A. Olson, *Journal*, Am. Concrete Inst., March–April, 1938, p. 401 (see especially Tables 2 and 4).

and F. G. Thomas.[30] The test results by Mr. Thomas are generally lower than the values computed from Eqs. 21 and 24*b* but higher than Eqs. 28 and 33*a*. This is consistent with his tests on axially loaded columns in the same series and the low strength appears to be due to the design of his test specimens. The reinforcing steel did not extend to the bearing plates but was embedded in enlarged ends which were smaller than those that proved unsatisfactory in the Illinois and Lehigh tests.[31] The test results indicated a slipping of the compression steel. The results of one series of tests are plotted in Fig. 12.

The results of the Richart and Olson tests[29] on 10-in. square columns also lie between the two sets of equations, but for some reason the axially loaded columns also failed to develop their expected strength. Messrs. Richart and Olson stated:[32]

> "Comparing the axially loaded columns of this series with previous tests in which 'flat-ended' columns were used, it appears that these 'hinged-ended' columns are weaker by roughly 15 per cent. It seems reasonable that the eccentrically loaded columns of this series are similarly weaker than columns rigidly framed into other members of a building frame."

Columns with Round Cores

The writer has derived formulas for the strength of round cored columns and they will be given herein, although further tests may show that the empirical constants can be improved.

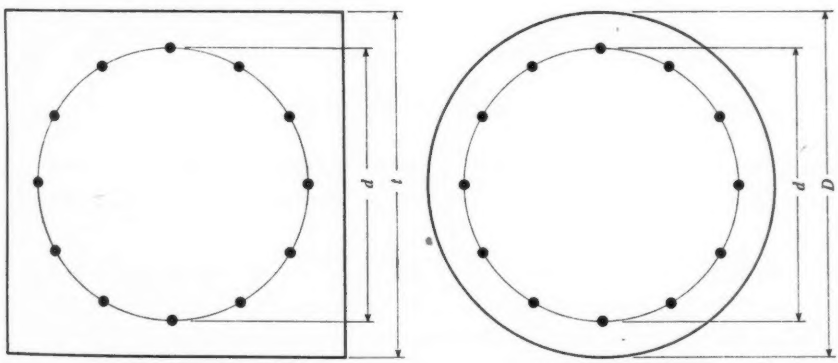

Fig. 13.—Columns with Round Cores

Square Columns.—When, in a square column (Fig. 13(a)), the longitudinal reinforcement is distributed around the circumference of a circle with a diameter d, Eqs. 28 and 33 can be adapted by substituting for the value of d' the equiva-

[30] "The Strength and Deformation of Reinforced Concrete Columns Under Combined Direct Stress and Bending," by F. G. Thomas, *Studies in Reinforced Concrete No. VI*, Building Research Station, London; see also, *Concrete and Constructional Engineering*, March, 1938, p. 165.

[31] "First Progress Report on Column Tests at Lehigh University," by W. A. Slater and Inge Lyse; and "Second Progress Report on Column Tests at the University of Illinois," by F. E. Richart and G. C. Staehle, *Journal*, Am. Concrete Inst., Vol. 27, 1931.

[32] "The Resistance of Reinforced Concrete Columns to Eccentric Loads," by F. E. Richart and T. A. Olson, *Journal*, Am. Concrete Inst., March–April, 1938, p. 406.

lent effective steel distance in terms of d. The equations seem to agree quite well with tests when 0.67 d is used, and assuming half the steel effective on each side of the section, Eq. 28 becomes, for ultimate load producing compression failure:

$$P = \frac{A_{st} f_s}{\frac{3e}{d} + 1} + \frac{0.85 A_c f_c'}{\frac{10.2 t e}{(t + 0.67 d)^2} + 1.51} \qquad (34a^*)$$

With a factor of safety of $2\frac{1}{2}$ Eq. 34a approaches $P = 0.4 A_{st} f_s + 0.225 A_c f_c'$ as e approaches zero. Making the same substitution in Eq. 33, it becomes, for ultimate load producing tension failure:

$$P = 0.85 t^2 f_c' \left[\sqrt{\left(\frac{e}{t} - 0.4\right)^2 + 0.67 \frac{d}{t} p_t m} - \left(\frac{e}{t} - 0.4\right) \right] \qquad (34b^*)$$

Round Columns.—To adapt Eq. 34a for compression failure to the case of the round column with round core, the effective depth of the concrete section, t, can be changed to 0.8 D and Eq. 34a becomes

$$P = \frac{A_{st} f_s}{\frac{3e}{d} + 1} + \frac{0.85 A_c f_c'}{\frac{8.16 D e}{(0.8 D + 0.67 d)^2} + 1.51} \qquad (35)$$

For the case of tension failure, a new equation can be derived in the same manner as Eq. 24b. For this purpose the distance from the center line of the section to the center of gravity of the area of concrete in compression can be expressed with sufficient accuracy as

$$\bar{x} = 0.211 D + 0.293 \left(0.785 D - \frac{2A}{D}\right) \qquad (36)$$

in which A is the compression area. Assuming that the total steel compression is equal to the total steel tension: $P = 0.85 A f_c'$; or, $A = \frac{P}{0.85 f_c'}$. Then $\bar{x} = 0.211 D + 0.293 \left(0.785 D - \frac{2P}{0.85 D f_c'}\right)$. Then assuming that four tenths of the total steel area is effective on each side, and that the effective d' is 0.75 d, adding 0.09 D to e to allow for deflection, and taking moments: $P (e + 0.09 D + 0.375 d) = (0.40 A_{st} \times 0.75 f_s d) + P (\bar{x} + 0.375 d)$ which reduces to the following equation for the ultimate load causing tension failure in a round column with a round core:

$$P = 0.85 D^2 f_c' \left[\sqrt{\left(\frac{0.85 e}{D} - 0.3\right)^2 + \frac{d p_t m}{2.5 D}} - \left(\frac{0.85 e}{D} - 0.3\right) \right] \qquad (37^*)$$

Although some of the assumptions used in deriving these equations may be questioned as being contrary to present conceptions based on the elastic theory, they may nevertheless be sufficiently close to the true conditions at failure so that the simple and convenient form of the equations will prove satisfactory in practice. A comparison[6] with the tests reported by Messrs. Richart and

Olson[29] indicates that this is true, although the writer feels that sufficient tests have not been made to show whether or not the empirical constants in the equations can be improved. The equations appear to be more convenient than any others that have yet been proposed.

Method of Design and Factor of Safety

The foregoing equations provide a simple and consistent treatment for the full range of conditions from axially loaded columns to beams under flexure alone. Designers who follow current methods are inconsistent in applying the plastic theory to axially loaded columns, and at the same time attempting to combine it with the theory of elasticity as soon as the eccentricity is introduced. Messrs. Richart and Olson[29] suggest that design of eccentrically loaded columns for the range of eccentricities with e less than $0.25\,t$ should be based on the transformed section modulus of the uncracked section, involving the modular ratio. They propose the use of the standard straight-line stress formulas for large eccentricities. For the allowable unit stress, they propose an empirical variation from $0.225\,f_c'$ for a concentric load to $0.4\,f_c'$ for larger eccentricities. Fig. 12 shows the relative strength computed by their method.

The method suggested by Thomas[30] using the present formulas based on the straight-line no-tension theory is similar to the method proposed by Messrs. Richart and Olson. Thomas introduces an empirically varying allowable concrete stress only in the range between $e = 0$ and $0.5\,d$. As applied to the full range of eccentricities such a method merely forms an empirical transition between the standard plastic treatment of the axially loaded column and the standard straight-line working stress treatment for beams, which is not as realistic nor as simple as the method proposed by the writer.

The equations of this paper, except Eq. 30, are ultimate strength equations. They are not intended to be used to predict stresses under working loads.

Actual stresses under working loads are affected by so many conditions that they are quite indeterminate. It has been proved[33] that shrinkage and plastic flow entirely upset stress computations based on the elastic theory but they have little effect on ultimate strength because before failure the strains are great enough to cause redistribution of stress.

This is also true of pre-stressed reinforcement which has been shown to have no appreciable effect on ultimate strength although it does increase the load at which tension cracks appear in the concrete.[34]

The factor of safety should obviously depend on the nature of the structure and its loading. It may be desirable to provide different factors for different kinds of loads in some cases. Reasonably well-controlled concrete is such a reliable material at present that the use of any factor of safety to provide for sub-standard concrete can be eliminated by using f_c' as the minimum cylinder strength instead of the mean. With care, sufficient concrete strength can readily be provided and the concrete can be placed on the same design basis as steel.

[33] "Effect of Time Yield in Concrete Upon Deformation Stresses in a Reinforced Concrete Arch Bridge," by W. M. Wilson, M. Am. Soc. C. E., and R. W. Kluge, Assoc. M. Am. Soc. C. E., *Bulletin No. 275*, Eng. Experiment Station, Univ. of Illinois, 1935.

[34] "Cracking in Reinforced Concrete," by F. G. Thomas, *The Structural Engineer*, July, 1936, p. 298.

If the minimum fatigue strength of reinforced concrete members, under load repeated thousands of times, is more than 50% of the ultimate static strength as indicated by tests, it will be still higher under loads repeated as infrequently as the maximum loading conditions in an ordinary structure. As a general proposition it appears that allowable dead plus live loads could be four tenths of the ultimate loads; and, for extreme combinations of dead and live loads, and wind, temperature, and shrinkage effects, one half of the ultimate might not be excessive. This is premised on the use of the elastic theory (and not the principle of moment redistribution based on yielding) to determine moments and thrusts so that the actual forces will not exceed the useful limit of the strength of the member. Each case should be given careful consideration and it is obvious that the designer should have as correct an understanding as possible of the true action of structures.

There is one important peculiarity of members subjected to both bending and direct load which has not been properly recognized. As can be seen from Fig. 12, when the tensile strength controls (Eq. 24*b*), the bending moment that can be resisted increases with the direct load. Therefore, it is possible for a reduction of direct load without change in moment to cause failure. It may not be safe, therefore, to consider that in case of overload, both the direct thrust and the moment will increase proportionately. It is obvious that in the case of a bridge arch, where the thrust is due largely to dead load and the bending moment due to live load, an increase in bending is much more probable than an increase in thrust. The use of ultimate strength formulas makes it possible to design a member for any combination of multiples of the thrust and moment. Instead of using a simple factor of safety as for an axially loaded column or a beam, in the case of a member subjected to both direct load and bending, consideration should be given to probable combinations to which the member might be subjected. A further analysis of the desirable factor of safety of arch ribs has been made by the writer.[23]

Comparison of Plastic Theory with the Standard Theory

The plastic theory proposed by the writer for the design of concrete members is based on three assumptions: (1) No tension in the concrete; (2) linear variation of strains; and (3) a simple equivalent stress block which closely approximates the effect of the real distribution at maximum load. It differs from the ordinary straight-line stress distribution theory only in that it recognizes the plastic action of concrete at failure instead of attempting to predict the stresses at working loads.

The objections that may be raised to the adoption of the plastic theory are about as follows: (*a*) It is unfamiliar and untried; (*b*) it is an ultimate strength method instead of a working stress method; (*c*) the present method has given satisfactory service for years and could be modified to permit higher stresses if they are justified; and (*d*) it may lead to the use of beams and slabs that are too shallow and cause excessive deflections.

Each of these ideas will be discussed in order, as follows:

(*a*) The application of the plastic theory is much simpler than the present method, and it is as well supported by tests as the latter. In fact, its closer

agreement with tests is one of its greatest advantages. It can be taught as easily as the present method and will lead designers to a better understanding of the actual structural action.

(*b*) In spite of the designer's familiarity with the working-stress method of design and his preference for it, the ultimate-strength method is superior if it is a better guide to the useful strength of members and provides a more consistent factor of safety against damaging overload. Tests indicate that the useful strength bears a more definite relation to the ultimate strength than to theoretical working stresses given by the standard formulas.

(*c*) The present method has been useful during a time when knowledge and experience with reinforced concrete were being accumulated. It provides, in some cases, what is now an excessively high factor of safety against concrete failure. It leads to uneconomical design and inconsistent factor of safety. It is actually based on an empirical value of $\frac{E_s}{E_c}$ which could be modified together with the allowable stresses to give practically the same results as the new theory as far as the resisting moment of the concrete in compression is concerned. This would lead to the use of nominal working stresses which would appear unreasonably high; but, more serious than that, as previously stated, the method does not give as satisfactory values of the steel lever arm as the new method. This would lead to the use of more steel in lightly reinforced beams and a reduction in factor of safety against steel failure in heavily reinforced beams as compared with the new method. Table 7 shows the average unit

TABLE 7.—Comparison of Cylinder Strength with Maximum Unit Stress in Over-Reinforced Beams

Item No.	Value of n in the formula	Ratio[a] $\frac{f}{f_c'}$	Mean variation from the average
	By the Straight-Line Formula:		
1	American Concrete Institute, 1928[b]	1.57	±0.045
2	Joint Committee, 1924[c]	1.50	±0.059
3	Actual	1.62	±0.075
	By the Parabolic Formula:		
4	American Concrete Institute, 1928[b]	1.10	±0.034
5	Joint Committee, 1924[c]	1.07	±0.041
6	Actual	1.05	±0.035
7	By the plastic theory, Eq. 12[d]	1.05	±0.037

[a] Computed unit stress divided by the cylinder strength.
[b] "Reinforced-Concrete Building Regulations and Specifications," *Proceedings*, Am. Concrete Inst., Vol. XXIV, 1928, Section 601.
[c] "Standard Specifications for Concrete and Reinforced Concrete," Report of Joint Committee, *Proceedings*, Am. Soc. C. E., October, 1924, Section 103.
[d] Ratio for Item 7 is the computed unit stress divided by 0.85 f_c'.

concrete stress, at failure, of the test beams reported by Messrs. Slater and Lyse,[15] as computed by seven different methods. The last column indicates the consistency with which the formula might predict the results of the tests if the correct unit stresses were used in the formula.

It is interesting to note that the straight-line formula with the actual value of n (Item 3, Table 7) gave the poorest results, and that the parabolic formula

(Item 6) and the plastic formula (Item 7) gave the best results. The plastic theory appears more desirable than a modified straight-line theory.

(*d*) The new formulas permit the use of more slender members, which is an advantage in structures for which dead weight and temperature or shrinkage moments are important. It will also be beneficial at the haunches of T-beams. It is always important to guard against excessive deflections, no matter what method is used, and it is always necessary to design carefully for shear and bond. It will not generally be found economical to use maximum percentages of steel in beams and slabs; but the designer will be given more freedom in design, and, having simpler tools with which to work, he can pay more attention to other important considerations.

APPENDIX

Description of Tests Plotted in Fig. 7

The tests by Messrs. Humphrey and Losse[18] were made on 333 8-in. by 11-in. beams of gravel, limestone, granite, and cinder concrete varying in strength from 1,500 to 5,700 lb per sq in. Reinforcement varied from 0.49% to 1.96% of steel with a yield point averaging about 40,000 lb per sq in. Of the total, 84 beams are not plotted in Fig. 7 because the concrete cylinder strength exceeds the testing machine capacity and, therefore, f_c' was not determined. The cylinders tested are 8 in. by 16 in. and f_c', the strength of 6-in. by 12-in. cylinders, is calculated as $\frac{100}{95}$ times the reported value. Each point plotted represents the average of three beams.

The U. S. Geological Survey Tests[35] were made at St. Louis, Mo., in 1908 on a series of 8-in. by 11-in. beams of gravel concrete with 3.02% of steel with an average yield point strength of 38,210 lb per sq in. Only beams with concrete strengths greater than 2,000 lb per sq in. are included, and because individual steel strengths are not reported, only the averages are plotted for two groups of eleven beams each, one under-reinforced and one over-reinforced. The concrete cylinders were 8 in. by 16 in. and f_c' is calculated to be $\frac{100}{95}$ times the reported strengths.

The Columbia University tests[36] were made on thirteen rectangular 12-in. by 13-in. beams reinforced with ordinary steel and twin twisted bars with yield points from 34,800 lb per sq in. to 58,250 lb per sq in. and areas varying from 0.451% to 1.245%. The concrete strengths varied from 2,510 to 3,443 lb per sq in. The yield point of the twin twisted bars was taken as the stress producing a unit strain of 0.004.

The tests by Messrs. Slater and Lyse[37] are those previously quoted. The four sets excluded from the averages in Table 1 are not plotted. Each point represents the average of a group of three beams.

[35] *Proceedings*, Am. Concrete Inst., Vol. 16, 1920, p. 127.

[36] *Concrete and Constructional Engineering*, March, 1937, p. 200; *Journal*, Am. Concrete Inst., November–December, 1936; and *Proceedings*, Am. Concrete Inst., Vol. 33, p. 183.

[37] *Journal*, Am. Concrete Inst., Vol. 33, p. 488.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

EXPANSION OF CONCRETE THROUGH REACTION BETWEEN CEMENT AND AGGREGATE

By Thomas E. Stanton,[1] M. Am. Soc. C. E.

Synopsis

Tests have demonstrated that excessive expansion of concrete may occur through chemical reactions between cements of relatively high alkali content and certain mineral constituents in some aggregates, such as certain types of shales, cherts, and impure limestones found along the coast of California between Monterey Bay on the north and Los Angeles County on the south. A new test procedure is described in this paper through which it is possible, in a comparatively short time, to develop the deleterious characteristics of cement-aggregate combinations similar to those reported in the California study. The procedure consists of curing the specimens in sealed containers at normal temperatures.

Introduction

Numerous tests conducted during recent years have established the value of low-heat cements in minimizing the shrinkage of concrete, and the value of the sulfate-resistant standard and puzzolanic-type cements in resisting attack by the sodium and magnesium sulfates and other deleterious constituents in alkali soils and sea water. Designers must now consider a third important factor in cement composition which may so affect the characteristics of a portland cement concrete as to produce excessive expansion and possible ultimate failure of a structure even if it is subjected to only normal curing and weathering conditions.

Given a properly designed cement-aggregate combination, such a condition can occur even when the cement is perfectly sound (in the generally accepted interpretation of the word) and of such a chemical composition as to be highly resistant to exterior attack by deleterious agents. This is fully and conclusively demonstrated through the tests described in this paper. A new test

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by April 15, 1941.

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procedure is also described without which it would probably have been impossible to make the laboratory tests.

Continual wetting or alternate wetting and drying of test specimens failed to develop (in any reasonable period of time, at least) any of the abnormal expansion characteristics experienced on many projects in certain districts of the state. It is now known that at least some of the field conditions of expansion can be duplicated in the laboratory when the specimens are cured in sealed containers, thereby retaining the original mixing water with few, if any, additions or subtractions through immersion or drying. It was not until this fact was discovered that invariable measurable expansion was obtained with certain cement-aggregate combinations, and that negligible, if any, expansion occurred with other combinations.

The complete documentary evidence sustaining the conclusions of the tests are presented for the first time in this paper, although a very brief preliminary report has been presented elsewhere.[2]

Salinas Valley Pavement Failure

Early in 1938, a section of concrete pavement north of the town of Bradley in the Salinas Valley, Monterey County, Calif., failed because of excessive expansion. An intensive field and laboratory investigation followed to determine the causes of the failure and to develop safeguards to prevent a recurrence of the condition in future work. The failed pavement was built in the fall, winter, and spring of 1936–1937. Most of it had passed through two winters when portions began to show distress in the spring of 1938. The distress became manifest in the form of excessive expansion which caused buckling at the expansion joints and a severe cracking throughout the length of certain slabs (Fig. 1(*a*)).

A coarse aggregate from a local commercial plant, located on a tributary to the Salinas River, was used throughout. The fine aggregate came from two sources, one the local deposit (herein referred to as Oro Fino), from which all of the coarse aggregate was secured, and the other (Coyote) imported from a commercially operated deposit near San Jose, Calif., in the San Francisco Bay area. The sands from these two deposits were used during alternate periods so that it was easy to identify the source of the fine aggregate in each section or day's run. A close inspection of all sections readily disclosed that excessive expansion had occurred only in those sections in which the local (Oro Fino) fine aggregate had been used, and it was obvious, therefore, that aggregates from this area contained at least one of the sources of trouble.

Earlier Failures

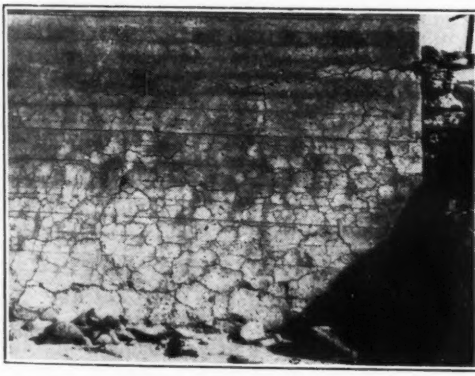
The Bradley pavement failure culminated a series of concrete failures along the coast area from Monterey County in the north to the northern part of Los Angeles County in the south, throughout which aggregates of the same typical characteristics are encountered.

The first structure in which distress developed to such an extent as to attract attention was the King City Bridge (Fig. 1(*e*)) built in 1919–1920

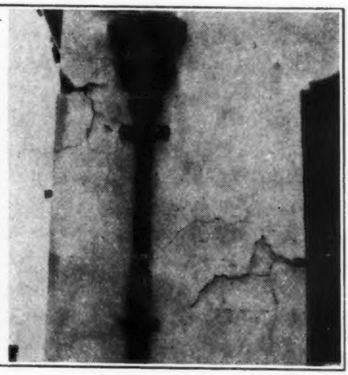
[2] "Influence of Cement and Aggregate on Concrete Expansion," by Thomas E. Stanton, *Engineering News-Record*, February 1, 1940, p. 59.

(*a*) Pavement

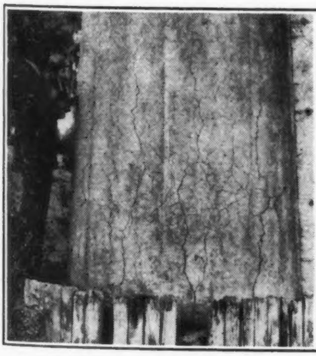
(*b*) Bridge Girder



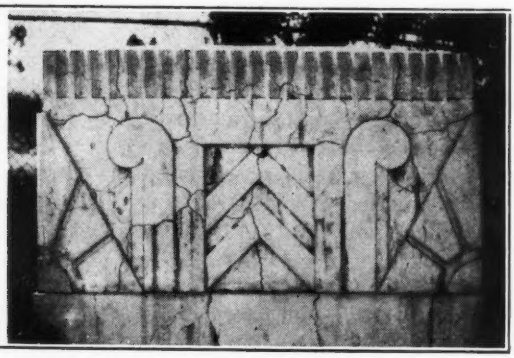
(*c*) Sea Wall



(*d*) School Building



(*e*) Bridge Pier



(*f*) Bridge Pylon

Fig. 1.—Typical Concrete Failures Resulting from Combinations of Certain Aggregates and High-Alkali Cements

across the Salinas River at King City, Monterey County. This bridge consists of an all-concrete trestle approach to a series of concrete piers and steel trusses over the main channel of the river. The channel structure consists of steel trusses resting on concrete piers of two circular sections with battered sides connected by a 12-in. diaphragm wall.

The coarse aggregate was imported from an unquestioned source, but the fine aggregate was taken from the bed of the Salinas River, or a small tributary (Pine Creek) emptying into the river a short distance upstream from the bridge.

Within three years after construction, well-defined cracks were in evidence in the caps of the piers. Later, these cracks extended into the columns of the piers. By July, 1924, all of the pier caps were more or less affected.

The concrete in the pile and trestle construction has shown little, if any, distress, the severe distress being confined to the piers. Although it is possible that the cement was not of a uniform quality, so far as is known the two concretes differed only in the sands used. Pine Creek sand was used at first; then sand from pits in the stream bed of the Salinas River and later surface sand from the bed of the same river, deposited during the winter rains, was used.

The condition of the King City Bridge has been investigated by a number of agencies, including representatives of the Bureau of Public Roads, the Bureau of Standards, the Portland Cement Association, and the cement companies, as well as engineers and chemists of the California Division of Highways. In general, the number of opinions expressed equaled the number of reports made. The causes of failure advanced by the different investigators were the sand; unsound cement; excess water; extreme variations in temperature; faulty curing; faulty concrete design; variable coefficient of expansion between different parts (including reinforcing steel); electrolysis; overstressing or under design; and rusting of anchor bolts, reinforcing steel, iron pipes, washers, etc. All of the assumed causes, however, were purely speculative without any substantiating foundation and (in 1940) have remained in the category of interesting theories.

The King City Bridge has been emphasized because it was the first partial failure of a major structure to attract attention and because the definite cause of the difficulty has been an unsolved mystery. Subsequent to the King City Bridge trouble, several concrete trestles, built afterward in Monterey County, in which fine aggregate from the Salinas River or its tributaries had been used, developed serious distress (Fig. 1(*b*)).

Similar trouble had been noted in Ventura County where the concrete piers of the Southern Pacific Main Line Bridge over the Santa Clara River near Montalvo, built in 1914–1915, had developed map cracking similar to the Salinas Valley structures. On June 5, 1933, George W. Rear, M. Am. Soc. C. E., bridge engineer for the Southern Pacific Railroad Company, wrote that the piers showing distress had been built in 1914, using local sand from the bed of the stream, whereas imported sand had been used the following year in constructing the piers which were still in good condition after eighteen years.

Other pavement and structure failures were noted in Los Angeles, Ventura, Santa Barbara, San Luis Obispo, and Monterey counties, where the aggregate was all of the same general type, containing from 4% to 15% total of shale,

cherty shale, and chert. A number of the more important failures were some sections of pavement and sea walls in Ventura and Santa Barbara counties (Fig. 1(*c*)).

Although the sulfates in sea water undoubtedly contributed to the serious deterioration of the sea walls, it was obvious that this deterioration was accelerated by the infiltration of sea water through cracks resulting from excessive expansion occasioned by other causes. The concrete walls of a school building in Santa Barbara, built in 1931, had cracked to such an extent by 1934 as to require at least partial reconstruction (Fig. 1(*d*)). All of the failures noted followed the same general pattern and were apparently due to the same underlying cause.

The solution of the problem was complicated by the fact that, although without exception cracking of the nature described was noted only when local fine aggregate had been used, there were numerous instances where the apparently same local aggregate performed in an entirely normal and satisfactory manner.

Immediately north of the pavement failure in Montery County is a section of pavement, constructed at least six years earlier, in which fine aggregate from the Salinas River was used, but which is still (1940) in excellent condition and shows no evidence of excessive expansion or cracking.

Development of Test Procedure

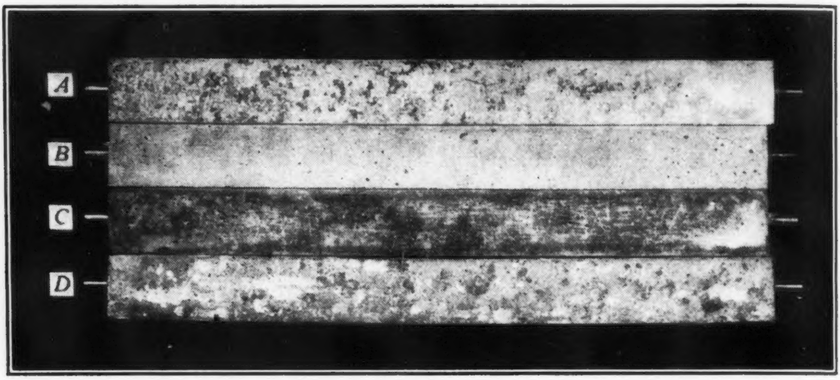
Upon conclusive evidence that the local sand used in the Bradley pavement, constructed in 1936–1937, had played an active part in the failure, a laboratory study was undertaken to determine, if possible, the cause or causes of the trouble and the reason for the inconsistencies that had been noted not only in the locality under immediate investigation, but likewise in all of the coast counties from Monterey to and including the northwesterly part of Los Angeles County.

The first step was to make up mortar and concrete bars of the local (Oro Fino) and the imported (Coyote) fine and coarse aggregates, and then subject some of these bars to continual wetting, some to continual dry exposure, under normal laboratory temperature and humidity conditions, and some to alternate wetting and drying. In one series, after initial curing for seven days, the specimens were dried for three days in an oven at 150° F and then soaked in water at 70° F for four days, the complete cycle taking one week. Another set was dried for two days in the oven at 150° F and soaked seven days in water at 160° F, the complete cycle requiring nine days.

Under none of these curing methods was any excessive expansion observable, even up to more than one year, regardless of the source or nature of the sand, except in the case of a coarse aggregate comprising 100% shale. In that case the expansion was obviously due to a physical expansion of the shale which, when present in large or concentrated quantities, had the effect, ultimately, of rupturing the concrete, but, when present in the normal percentages found in the aggregate under suspicion, had no measurable deleterious effects for the period under observation. In the case of the coarse aggregate with 100% shale, failure occurred on the fifty-first cycle of wetting and drying, at which time the expansion was 0.193%.

An Accidental Discovery

In the meantime, however, upon removing the cover from a 2-in. by 4-in. mortar cylinder which had been cast and retained in a tin container for one year, the specimen was observed to be covered with blotches fringed with a



(*a*) Relative Length of 1-in. by 1-in. by 10-in. Mortar Bars (Mix 1 : 3) Cured for Four Months Under Various Conditions

(*b*) Mortar Specimens (Mix 1 : 2) After Curing Seven Months in Sealed Containers

Fig. 2.—Condition of Laboratory Specimens After Curing in Air, Water, and Sealed Containers

white efflorescence (subsequently analyzed as sodium carbonate), and in a short time the entire specimen became covered with cracks similar to those shown in Fig. 2(*b*) and also to the crazing noted in the pavement and other structures in the area under investigation. A similar condition will be noted in

the case of the mortar bars in Fig. 2(*a*). These specimens are identified, by letters, as follows:

Specimen (see Fig. 2(a))	Curing method	Expansion after four months, in millionths of an in. per in.
A	Sealed container	+7,310
B	Air	−1,350
C	Water	+ 40
D	Sealed container	+7,230

All bars contain the same quantity of high-alkali cement (1.14%) and a neutral aggregate to which was added 10% of the deleterious mineral No. 28039 (see Appendix I).

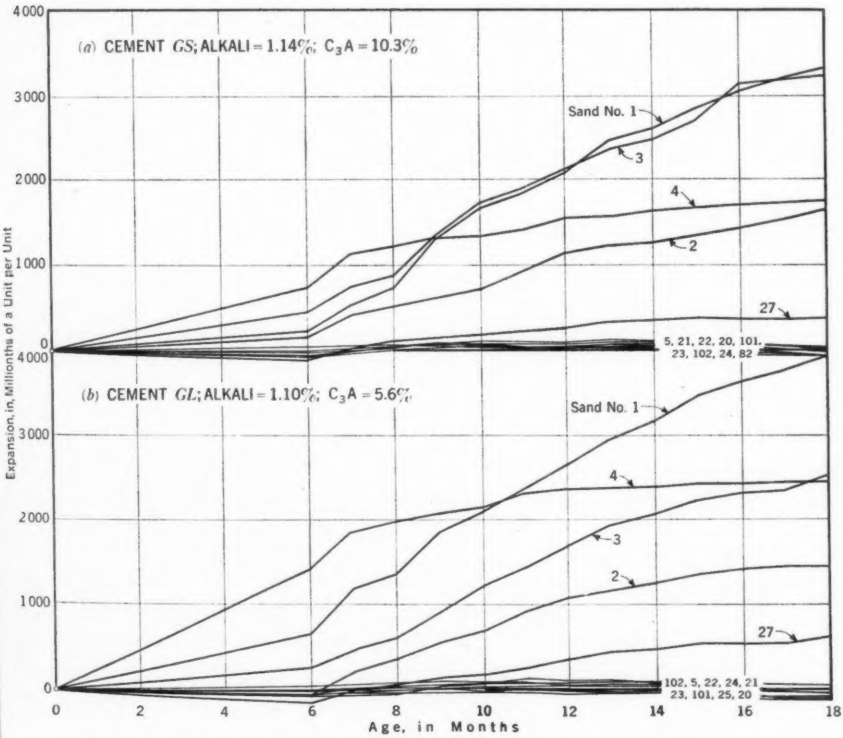


Fig. 3.—Expansion of Mortars on Certain California Sands and High-Alkali Cements of Relatively High and Low C_3A-Content

It was apparent that, when test specimens were kept in sealed containers, or at least protected from the atmosphere and any drying out, but at the same time prevented from any leaching of salts by constant immersion or alternate wetting and drying, a chemical reaction was going on which caused an excessive expansion of the mortar, with ultimate failure.

A series of tests was then started on 1-in. by 1-in. by 10-in. mortar bars cured in sealed containers, as well as in water, and measured for expansion at different ages. Experience with hundreds of specimens, subsequently fabricated, has invariably checked the results accidently noted in the case of the 2-in. by 4-in. mortar specimen. Bars fabricated with the same sands and the same cement have invariably followed the same expansion pattern.

The local Oro Fino fine aggregate (sand No. 4, Appendix II, and Fig. 3) used in the Monterey County job, in combination with the type of cement used in the work, always shows excessive expansion in a few months (considerable even at twenty-eight days); but the imported (Coyote) sand (No. 20) with the same cement has shown negligible expansion in periods exceeding eighteen months.

In Fig. 3, the specimens were of a 1 : 3 plastic mortar mix, and were cured in sealed containers. The C_3A-content (tricalcium aluminate) does not appear to be a factor in the expansion reactions. Later tests on specimens stored in the moist room, but protected from excess moisture, indicate the same expansion reaction as in sealed containers. However, for the sake of uniformity and convenience, all specimens, other than those constantly wet or dry, were cured in airtight cans.

Curing Procedure

Immediately following fabrication, the test bars are cured twenty-four hours in the moist room under standard conditions. They are then removed from the molds, identified by marking with an indelible pencil, measured, and immediately placed in the containers.

The containers used are regular 6-in. by 12-in. tin cylinder molds with the joints soldered to prevent loss of moisture. The slip cover is probably sufficiently tight to seal the specimens without any other precautions, but if there is any doubt a strip of adhesive tape is wrapped around the joint.

Only enough free water (30 cc to 50 cc) is placed in the bottom of the container to insure keeping the air humid. The specimen rests on the ½-in. monel metal gage point, and therefore the mortar of the bar is not in direct contact with the water.

Eighteen 1-in. by 1-in. by 10-in. specimens (a day's run) are usually stored in one container. The containers are stored under ordinary room temperature, no special effort being made to keep the temperature constant. The specimens are removed from time to time for measurement and then immediately returned to the containers for further curing and later tests. If rusted badly, the containers are replaced.

Cement

Other concrete structures in this area, in which the same type of local sand but a different cement had been used, developed no distress up to five or six years at least. This fact gave rise to the suspicion that the expansion resulted from a chemical reaction between certain ingredients in the aggregate and some ingredients in the cement. Alkali in the cement in the form of the sodium and potassium oxides was suspected and, as the cement used on the Bradley job

contained approximately 1.14% total alkalis, the tests were repeated with another cement low in alkali (0.45%), with definite results, in that there was little, if any, expansion with the low-alkali cement in combination with the local (Oro Fino) sand which had shown so much expansion with the 1.14% alkali cement. The tests have been repeated many times, not only with the two aforementioned cements, but with other California cements of intermediate alkali content and sands from within and without the coast area.

The resultant expansion has almost invariably been proportional to the alkali content of the cement, whenever used in combination with aggregates containing minerals of the deleterious type found in the area under invesitgation. On the other hand, mortars containing commercial sands originating outside of this area, and containing negligible percentages of shale and chert, being essentially of quartz and feldspar fragments with some granitic and sandstone particles, have developed little, if any expansion, even with high alkali cements.

The data in Figs. 3 and 4 are offered in support of this observation, the mix in each case being a 1 : 3 plastic mortar, and the specimens all being cured in sealed containers. Note that cement *HP* is a puzzolanic type.

Tests on sands Nos. 2 and 4 made after those in Fig. 3 showed expansion as great as, if not greater than, sands Nos. 1 and 3. It would seem, therefore, that the tests in Fig. 3 were influenced either by lack of sufficient moisture to continue the reaction, or by a lower percentage of the deleterious ingredients than in later samples. In fact, these two conditions are probably the cause of some lack of consistency in the tests on other sands, as uniformity of sampling is exceedingly difficult where trouble-causing ingredients are present in such small quantities.

When sand No. 5, Fig. 3, failed to develop excessive expansion, a mineralogical study was made, and it was found that this sample of sand contained negligible percentages of shale, chert, and impure limestone. Therefore, the results are consistent with the nature of the material and confirm the conclusion previously reached regarding the probable source of the trouble.

Within the area under investigation, all of the aggregates from commercial sources, with one possible exception, contain appreciable percentages of shale and chert. All sands with small percentages, or entirely free, of shale and chert show little, if any, expansion up to one year, regardless of the cement used. All sands in the coast area relatively high in shale and chert, on the other hand, develop expansions in a few months roughly in proportion to the alkali content of the cement. In the latter case, negligible expansion is observed in the case of cements with less than 0.5% alkali and high expansion in the case of the 1.14% alkali cement, the other cements ranging in between roughly in proportion to the alkali content (see Fig. 4). Some inconsistencies are noted, but not of such magnitude that they cannot be accounted for by variations in the aggregate.

It is obvious, therefore, that in the presence of certain minerals an otherwise sound but high-alkali cement is a contributing factor to the excessive expansion and subsequent failure of a concrete structure. Under certain conditions this expansion may not be sufficient to disrupt the structure and may cease as soon as the reaction has been completed in any particular cement-

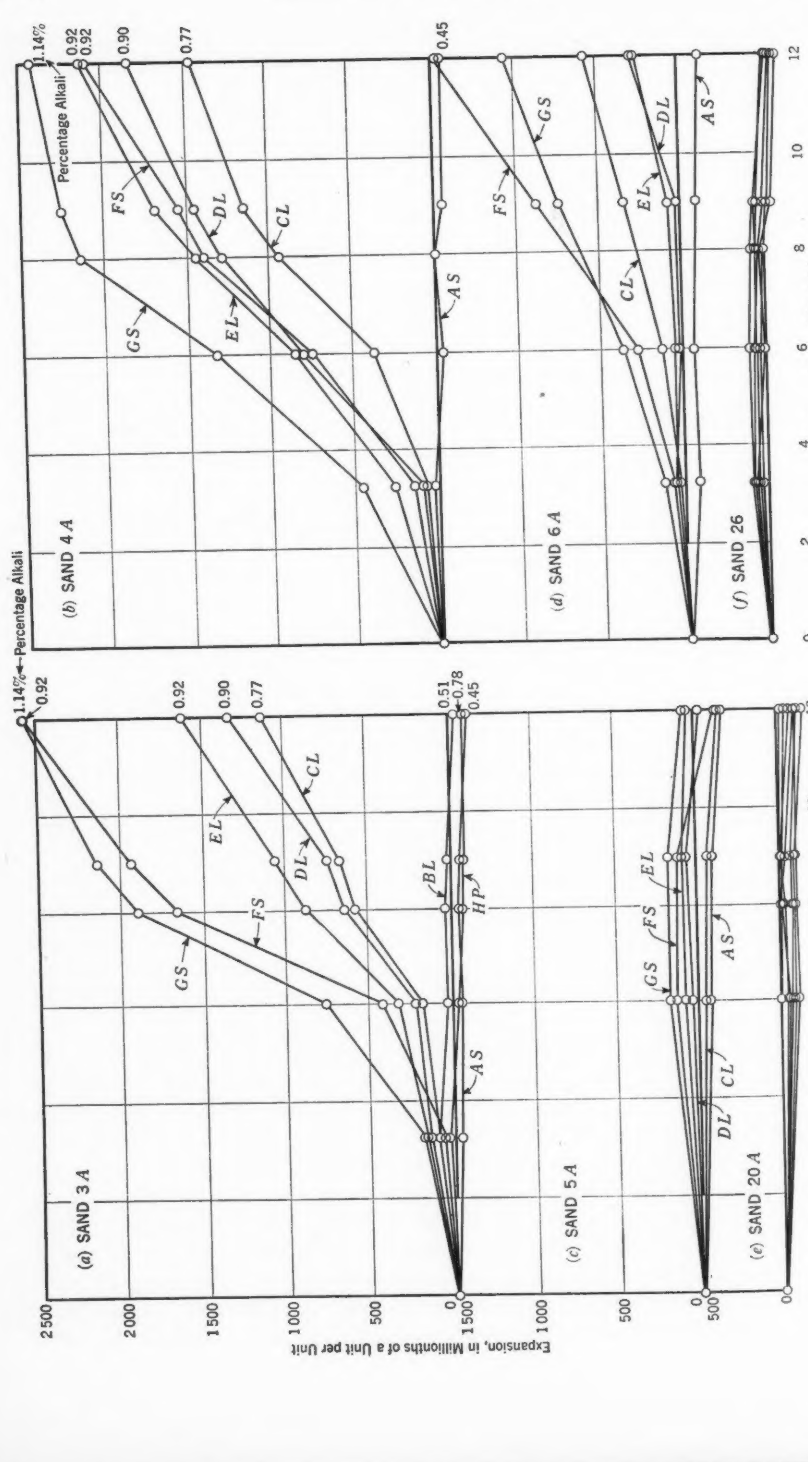


FIG. 4.—EXPANSION OF SAND-CEMENT MORTARS AS INFLUENCED BY THE TYPE OF SAND AND THE ALKALI CONTENT OF THE CEMENT

aggregate combination. In other cases, however, the initial cracking, although above normal but not excessive, may have very serious consequences when the structure is subsequently subjected to attack by sea or alkali waters which, seeping into the concrete mass through fine cracks formed from the primary cause, may take up the work of destruction through corrosion of reinforcing steel or attack on the vulnerable compounds of the cement itself.

Aggregates

The coast aggregates included in this study consist of mineral particles predominantly granitic in character with some quartz-feldspar, sandstone, impure limestone, shale, and chert. The granite, quartz-feldspar, and sandstone usually comprise more than 90% of the total; the impure limestone, shale, and chert comprise the remaining 5 to 10%. None of the adverse reactions have been traced to the first group. Some of the minerals in the second group, however, are definitely reactive under appropriate weathering conditions. The minerals in this latter group originate in the Upper Miocene sedimentary deposits. According to Ralph D. Reed,[3]

> "The Upper Miocene in California was a period of widespread seas in which deposition of organic siliceous shale of the Monterey type took place * * *. Whatever conditions are necessary for the deposition of this material prevailed widely during the Upper Miocene from Lower California at least to the Northern Coast Range province and from the San Joaquin Valley area to the present shoreline of the Pacific Ocean."

Many of the types of shale and chert found in the Upper Miocene formations of California are different in several respects from the shale and chert of other geological periods commonly found in other parts of this state and the United States. It has been clearly demonstrated by geologists that the Monterey series of the Miocene, and particularly the Modelo formation, is unique in regard to petrological classification and mineralogical content. Although most of the Modelo formation consists largely of diatomaceous, tuffaceous, and other siliceous shales, many lenses and thin layers of calcareous shale ranging from a fraction of an inch in thickness to a number of feet are intermixed throughout the strata. It is common to find lime, opaline silica, or some chalcedonic silica as the cementing material in the shale. In many cases opaline silica and chalcedony are the main minerals in thin lenses and layers. The opaline silica greatly predominates over chalcedony, but these silicified bands, layers, and concretions have nevertheless been classified as chert by geologists and other authorities on the petrography of the Miocene formations.

The California cherts may be divided into two main classifications: (1) The deleterious cherts of the opaline types, which may or may not contain some chalcedony; and (2) the less questionable group of quartzose particles which includes the red and green varieties of chalcedonic silica such as radiolarian chert of the Franciscan formations, described and named by A. C. Lawson,[4] as well as jasper and the other varieties of chalcedony. The deleterious particles classified as chert contain opaline silica either in the main or as a

[3] "Geology of California," by Ralph D. Reed, 1933, p. 188.

[4] Geological Atlas, U. S. Geological Survey, San Francisco, Folio 193, 1914.

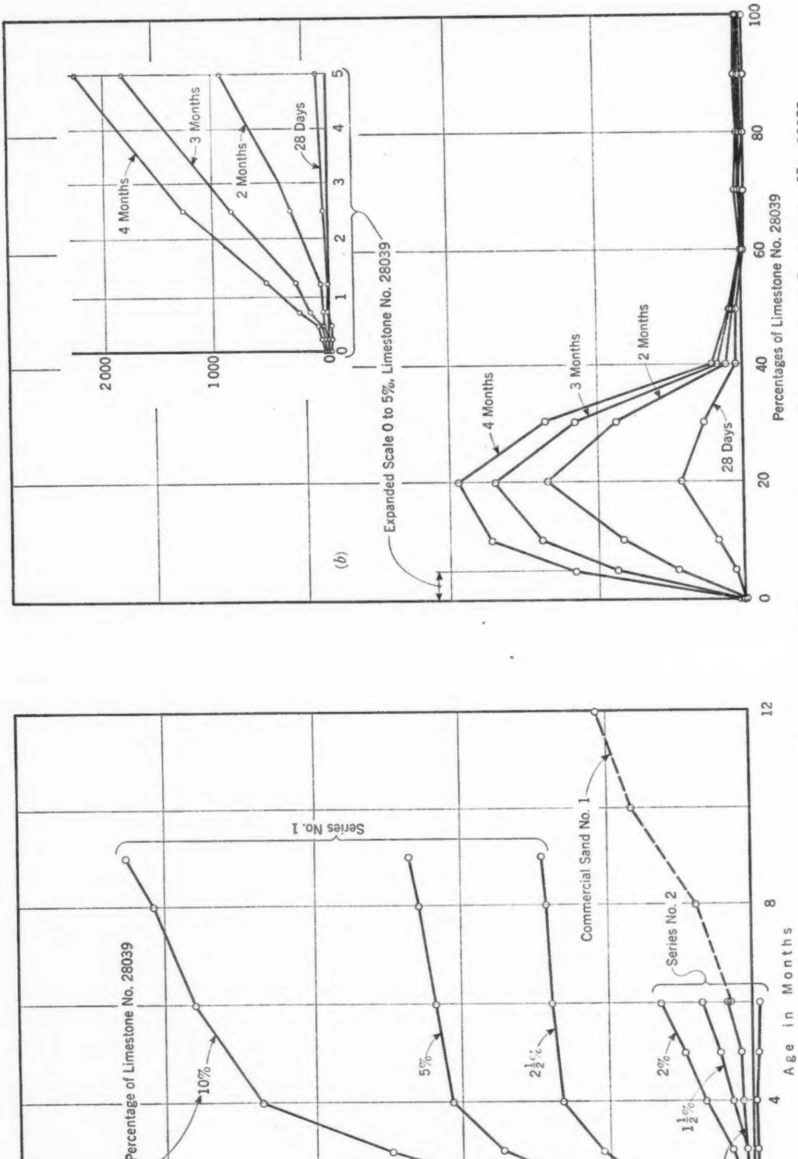


FIG. 5.—COMPARATIVE EXPANSION OF MORTARS CONTAINING SAND No. 21, IN RELATION TO AGE AND PERCENTAGE OF LIMESTONE No. 28039

cementing material throughout a shale particle. In the case of particles classified as cherty shale, some of the laminæ contain little or no opaline silica.

Throughout the coast area under investigation, the shale, chert, and impure limestone deposits range from types with negligible percentages of lime and magnesia to a high-lime magnesia shale which in this report is referred to as "siliceous magnesian limestone." This classification is consistent with that given by William Twenhoffel,[5] who states that:

> "The per cent of calcium carbonate or the double carbonate of calcium and magnesium in sedimentary rock ranges from nothing to approximately 100. If the per cent equals or exceeds 50, the rock may be termed a limestone. If the per cent is below 50 the rock should be assigned to some other group. There is, however, no sharp division between limestones and other sedimentary rocks; they grade without sharp break into the sandstones and shales, as well as nearly every other variety of rock."

Rocks covering a wide range from zero to a comparatively high percentage of lime and magnesia have been found in the coast area, and a sufficient number of these rocks have been included in the present study to cover the field fully.

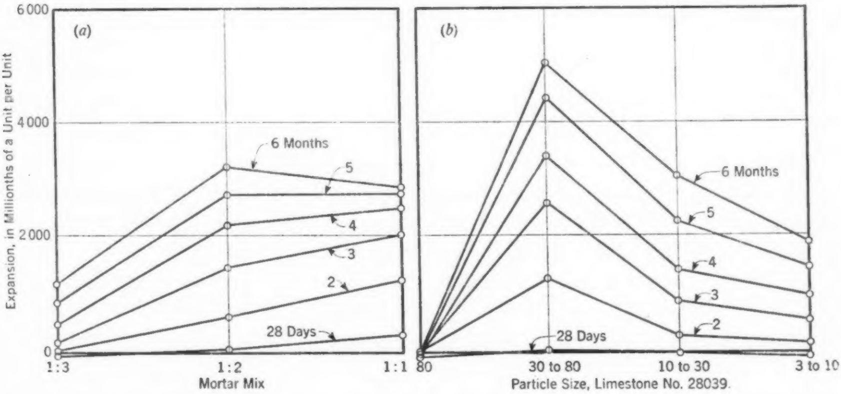


FIG. 6.—COMPARATIVE EXPANSION OF MORTARS CONTAINING SAND NO. 21, IN RELATION TO MORTAR MIX AND PARTIAL SIZE, THE PERCENTAGE OF LIMESTONE NO. 28039 REMAINING CONSTANT AT 5%

An extensive study has been made in an effort to isolate the "bad actors" in the mineral content. Considerable success has attended this effort. Although it appears very probable that some form of hydrous silica plays a very active part, the greatest reaction in the tests described herein has been with material classified as a siliceous magnesian limestone (Nos. 28039 and 28039*A* in Table 4, Appendix I), of relatively high specific gravity, low absorption, high sodium sulfate resistance, and low rattler loss. This mineral is one of the most active causes of the trouble, even when present in percentages less than 1.00 (see Figs. 5 and 6). The high-alkali cement *GS* (Fig. 3) was used throughout. The same aggregate with the low-alkali cement developed no expansion. On the other hand, several of the shales and cherts which expanded moderately in

[5] "Treatise on Sedimentation," by William Twenhoffel, 1932, p. 238.

water showed negligible expansion in sealed containers, even with a high-alkali cement (see Table 1), as contrasted with the siliceous magnesian limestone (No. 28039), which developed no expansion when cured in water but high expansion in sealed containers. It has not been determined if the reaction in the

TABLE 1.—COMPARATIVE EXPANSION OF MORTARS OF SAND NO. 21 WITH 10% OF DIFFERENT TYPES OF CRUSHED SHALE CHERT, AND SILICEOUS MAGNESIAN LIMESTONE AND A HIGH-ALKALI CEMENT

(In Millionths of a Unit per Unit)

Age	Mix[a]							
	1 : 3					1 : 2		
	No. 19374	No. 28037	No. 28038	No. 28039	No. 28042	No. 28044	No. 28045	No. 28046
28 days	+90	....	....	....	....	+55	+ 3	+ 25
2 months	....	....	....	....	....	+90	+42	+190
3 months	+238	+18	+47	+1,910	+62	+90	+370	+1,030
6 months	+1,220	− 5	+45	+4,140	+80	....	....	....
8 months	+2,065	− 3	+35	+5,175	+70	....	....	....
9 months	....	−38	− 3	+5,557	+72	....	....	....
1 year	....	−90	−37	+6,195	+50	....	....	....

[a] See Appendix I; all these specimens were cured in sealed containers.

case of this impure limestone is due entirely to some form of silica or in part to the magnesium carbonate. If it is due in any part to the magnesium carbonate, the probable chemical reactions in this respect are as follows:

The sodium oxide in the cement hydrates to form sodium hydroxide. The sodium hydroxide, in turn, probably reacts with at least some form of the magnesium carbonate in the siliceous magnesian limestone to form magnesium hydroxide and sodium carbonate; thus:

$$Mg\,CO_3 + 2\,Na\,OH \rightarrow Mg(OH)_2 + Na_2CO_3 \qquad (1)$$

The sodium carbonate ultimately crystallizes with seven to ten molecules of water, with an increase in volume, and thus causes expansion of considerable magnitude, resulting in stresses sufficient to rupture the concrete. The approximate volume change may be computed theoretically from Eq. 1, assuming the following specific gravities:

Description	Specific gravity
Magnesium carbonate in the rock	3.037
Sodium hydroxide	2.130
Magnesian hydroxide	2.300
Sodium carbonate including 10 molecules of H_2O	1.460

If the molecular weight of the reacting components and the end products are divided by the specific gravities, the following respective volumes are

obtained:

$$\left.\begin{array}{l} Mg\,CO_3 = \dfrac{\text{Molecular weight}}{\text{Specific gravity}} = \dfrac{84.33}{3.037} = 27.7 \\ \quad + \\ 2\,Na\,OH = \dfrac{80.016}{2.13} = 37.6 \end{array}\right\} 65.3 \text{ (Combined volume of reacting components)}$$

reacts to form:

$$\left.\begin{array}{l} Mg(OH)_2 = \dfrac{58.336}{2.3} = 25.36 \\ \quad + \\ Na_2CO_3,\ 10\,H_2O = \dfrac{286.17}{1.46} = 196.00 \end{array}\right\} 221.36 \text{ (Combined volume of reaction products)}$$

or an increase in volume of 239%.

To check the accuracy of this assumption, tests were made to determine the action of sodium hydroxide upon various materials that might be present in the aggregate, such as magnesium carbonate, calcite, dolomite, etc., as well as on the siliceous magnesian limestone.

The method used was to subject the substance under observation to the action of a normal sodium hydroxide solution for seventy-two hours, excluding the possibility of contamination by atmospheric carbon dioxide. A portion of the clear liquid was titrated with normal hydrochloric acid using methyl orange as an indicator. A second volume equal to that used in the methyl orange titration was treated with barium chloride solution to precipitate the carbonates and then titrated with normal hydrochloric acid with phenolphthalein as the indicator. The difference between the volumes of acid used in these determinations was computed to sodium carbonate.

Treated in this manner a sample of:

(*a*) Standard calcite yielded only a trace of sodium carbonate as Na_2CO_3;
(*b*) Bakers calcium carbonate likewise yielded only a trace;
(*c*) Dolomite with 7.1% Mg O yielded 1.06% Na_2CO_3;
(*d*) Technical hydrated magnesium carbonate yielded 51.00% Na_2CO_3; and
(*e*) The siliceous magnesian limestone with 15.5% Mg O yielded 8.65% Na_2CO_3.

All material was crushed to pass an 80-mesh screen before test.

These results indicated that sodium hydroxide reacts with certain forms of magnesium carbonate, but not with carbonate of lime. It appears that some similar reaction occurs in the case of at least some of the hydrous silicates. Although some early expansion due to physical causes may be expected from some of the shales, and therefore the percentage of such shales should be held to a reasonable minimum (controlled by the sodium or magnesium sulfate test), nevertheless, when held to comparatively low limits, the expansive force may not be sufficient to cause rupture except in the nature of popouts when particles are close to the surface. This conclusion is confirmed by observation of numerous structures which, except for surface popouts, show no distress over a

period of years even if the aggregate contains a relatively high percentage of many of the types of shale found in the area under investigation. With some cherts and shales, however, in addition to the siliceous magnesian limestone, a chemical reaction of a decidedly destructive nature always takes place in the presence of a high-alkali cement.

An astonishing development was the effect of a very small percentage of the high lime-magnesia rock. As will be seen by reference to Fig. 5(*a*), as low as 1.00% of this mineral caused considerable expansion in six months, the expansion in that period being almost identical with the expansion mortars of sand No. 1 in the same period of time, indicating that sand No. 1 contains not more than 1.00% of deleterious particles but that even this low percentage is objectionable.

The particle size of this mineral also has an important bearing on the result. It was thought that, by crushing the deleterious particles to pass 200-mesh completely, an accelerated reaction might be had. The reverse was the case, as specimens fabricated with −80-mesh particles developed no expansion in sealed containers, whereas the −30-mesh to +80-mesh particle specimens showed the greatest expansion (see Fig. 6(*b*)). Therefore, it appears that the reaction between the reactive ingredient in the aggregate and the alkali in the cement, when the aggregate is in a finely divided state, is either dissipated throughout the mass in such a way as to cause no high expansive forces or the reaction is largely over before the concrete attains permanent set. One theory is that the expansion is taken up in the interstices and by the gel, thus reducing the measurable manifestation of the reaction to negligible proportions.

Merriman Alkalinity and Free Alkali Test

The Merriman test[6] for alkalinity and free alkali was applied to the cements, but it was found that this test was not always consistent with the results. One cement, relatively low in the Merriman test, but with high alkali as determined by chemical analysis, developed high expansion; and its action, therefore, was consistent with the total alkali as determined from the chemical analysis rather than with the percentage of soluble alkali as determined by the Merriman test. Another cement with a high Merriman test, but low in alkali as determined by the chemical analysis, gave low expansion results. The reason for the discrepancy appears to be that the soluble portions of some cements go into solution much more slowly than others. The conclusion has been reached, therefore, that the Merriman test procedure, in its present form at least, is not suitable for a proper classification of cements in the order of the reactions encountered in the California tests.

Extent of Tests

Tests have been made on fine aggregate from all of the commercial sources within the region affected and from several sources outside of this area for comparison purposes; they have been made also on most of the commercial cements manufactured in California ranging in alkali content from less than 0.50% to approximately 1.14% (Table 5, Appendix III).

[6] Specifications for Cement, Board of Water Supply, City of New York.

As previously stated, negligible expansion developed with sands from outside the affected area. With one exception, all of the local sands showed high expansion in proportion to the alkali content of the cement. With the exception of the one sand, all of the samples contained from 5% to 10% of shale, chert, and the siliceous magnesian limestone. The sample of sand (sand No. 5) from the one deposit which failed to react was subsequently analyzed and found to contain only a very small percentage of the deleterious particles.

Numerous combinations have been made of aggregate from all commercial deposits and cement from most of the California mills. For check purposes, the study has been expanded to include, also, several cements from other parts of the United States.

Following a visit to the laboratory, when he was in California during the summer of 1939, the late Thaddeus Merriman, M. Am. Soc. C. E., who was much interested in the study, forwarded a sample of low-alkali cement (0.47%) from a Lehigh Valley plant (cement *N* in Table 5, Appendix III); and Ira Paul of the New York State Highway Department furnished a sample of 0.64% alkali cement from a New York plant (cement *R*). Through F. V. Reagel of the Missouri State Highway Department and W. E. Gibson, Assoc. M. Am. Soc. C. E., of Kansas, cements were secured similar to those used in pavements in Missouri and Kansas (cements *M* and *O*). These out-of-state cements have all been tested in combination with the Oro Fino and Coyote sands.

The expansion reactions have paralleled the reactions of the California brands. Cement *M* with a high-alkali content (0.83%) has developed excessive expansion with the Oro Fino sand (4,010 millionths of a unit per unit in fourteen months) and negligible expansion with the Coyote sand, whereas cements *N*, *O*, and *R*, comparatively low in alkali, have developed little if any expansion with either the Oro Fino or Coyote sands.

More than 3,000 1-in. by 1-in. by 10-in. mortar specimens of all combinations were fabricated and tested for expansion under different curing methods. The present status of knowledge of the subject was only arrived at through following numerous leads, some of which led into blind alleys and others of which were productive of results pointing the way and suggesting additional tests or lines of investigation.

Series I.—This series was started in May, 1938, and included only the two sands used on the Bradley job. Several coarse aggregates were used, including chert, shale, granite, and an altered dacite.

The cements in this first series were of the brand used in the Bradley job (a high-alkali cement) and in addition a low-alkali cement from another mill. The test specimens consisted of 2-in. by 2-in. by $11\frac{1}{4}$-in. bars of six-sack concrete, using 50% sand and 50% coarse aggregate, and the maximum size was $\frac{3}{4}$ in. One set of bars was cured in water, one set in air, and other sets alternately soaked and dried at air and water temperatures ranging from 70° F to 160° F. These tests produced no results of value, even up to one year, and at no time duplicated field experience with the local fine aggregate and job cement.

All specimens cured in water showed a slight expansion in the earlier period, but by 200 days all showed shrinkage from the peak length. At one year all specimens were shorter than the original length with the exception of the shale

coarse aggregate specimens (ledge shale from Oro Fino Canyon), which showed a slight expansion (0.009%).

All specimens cured in air at laboratory temperature showed shrinkages ranging from 0.1% to 0.3%, but no cracking. In the alternately wet and dry group, there was shrinkage or negligible expansion with the exception of the coarse aggregate specimen of 100% shale which, by the fifty-first cycle, had expanded 0.193% and failed by cracking.

Series IA.—Shortly after Series I was started, supplementary specimens of the same size were placed under similar test, using aggregates from various spots in the Oro Fino deposit, and also from other commercial sources throughout the district between Monterey and Los Angeles counties. Again the results were negative.

Series II.—After Series I and IA had been under way approximately three months, the condition of the 2-in. by 4-in. mortar specimens cured in a sealed container was observed, and a new series was immediately started, the specimens in this series being fabricated in September and October, 1938. Because of the expense and labor involved in fabricating and measuring concrete specimens, and because it was apparent from an inspection of the mortar specimen cured in the sealed container that the fine aggregate was considerably more reactive, at early stages at least, than the coarse aggregate, Series II consisted of fine-aggregate mortar specimens only, and the specimens in all subsequent series, except one, were mortar specimens. At the start, six 1-in. by 1-in. by 10-in. 1 : 3 mortar bars were fabricated from each mix for curing under different conditions, and the results reported are the average of several specimens. As the 1 : 2 mortars were found much more reactive, all later specimens were fabricated from this mix.

Sands from fourteen sources were included in Series II—five from commercial sources throughout the affected area; seven from outside sources, free of shale and chert; and in addition a crushed diorite and a crushed quartz. With little information at that time relative to the most probable source of the trouble, and to ascertain if the situation could be corrected by the use of a low C_3A cement, two types of the brand of cement used on the Bradley job were included: (1) The job cement with 10.3% C_3A and 1.14% alkali, and (2) a modified cement from the same mill containing 5.6% C_3A and 1.10% alkali. To hold the amount of testing to a minimum, no other cements were included in Series II.

The mixes were put up in duplicate, one set with the high-alkali and comparatively high C_3A, and the other with the high-alkali and comparatively low C_3A cement. A uniform water-cement ratio (0.94) was used, the slump varying $1\frac{3}{4}$ in. to $2\frac{1}{8}$ in. with the natural sands, 1 in. to $1\frac{1}{4}$ in. for sands having rock chips in the coarser sizes, and $\frac{5}{8}$ in. to $1\frac{1}{4}$ in. for the crushed quartz and diorite. Some of the specimens were cured continuously in water, some in air, some alternately wet and dry, and some in sealed containers in the presence of a slight amount of free water.

As no material reaction was expected short of six months, the sealed containers were not opened or the specimens measured until six months had elapsed. The reaction was found to be so extensive by this time (Fig. 3) that in all

subsequent series the measurements were started at earlier periods. After measuring, the specimens were returned to the sealed containers for further curing and measurement at subsequent test periods, usually monthly. At six months all of the sands from the shale-chert infected area, except one, showed considerable expansion. The exception was the sand which was subsequently found to be low in shale and chert.

All of the sands from outside the infected area, except one (all of which have a good construction history), showed either only a slight expansion or a shrinkage at six months. This relative condition has persisted throughout the entire test period to date. The expansion of the one exception (No. 27 in Fig. 3) was not especially high, but, as it was decidedly out of line with the others, a thorough study is being made (1940) to ascertain the cause.

Mortar cylinders (2 in. by 4 in.) for compression tests, fabricated from the same mixes as the bars and cured in water, developed equal strengths up to one year for the poor and the good sands. Later specimens cured in sealed containers showed a gradual falling off in strength with age whenever the infected sands were used.

Series III.—As soon as Series II was well under way and time permitted, a third series was started similar to Series II, but including a number of other California brands of cement ranging from low to high alkali. In addition to the bars, 2-in. by 4-in. mortar cylinders were cast and cured in sealed containers for breaking at twenty-eight days, six months, and one, two, and five years.

As in the previous series, all bars cured continuously in water expanded slightly at first and then began to shrink. The first expansion measurements of the specimens cured in sealed containers were made at one hundred days. Even at that period, considerable expansion was noted in the case of the infected sands.

Series IV.—Having determined definitely from Series II and III that certain sands were reactive, the next step was to determine, if possible, which mineral ingredient or ingredients in the sands were the source of the trouble. Since all minerals except the shale and chert had been removed from suspicion by previous tests on selected sands, four suspected minerals were selected from the infected area, including a diatomaceous shale (No. 28037); a chert (No. 28038); and a calcareous shale (No. 28039), later classified as a siliceous magnesian limestone (Table 4, Appendix I). These shales and cherts were crushed to pass a No. 3 screen and were combined in various percentages with a neutral sand (No. 21) from the Russian River, north of San Francisco Bay.

This series was later expanded to include a serpentine found in the coast area, a calcareous shale (No. 19374) with approximately 6% CaO but low magnesia, a siliceous chert (No. 19375), and two opaline cherty shales (Nos. 28045 and 28046). An opal and an opalized chert were likewise included to determine the effect of an opaline silica. Crushed limestone was used in other specimens.

The conclusion from this test was that the principal ingredient in the sand contributing to the abnormal volume change was the "siliceous magnesian limestone" and that probably the magnesia in the form of $MgCO_3$ in this rock was one of the active reagents with the sodium hydroxide from the sodium oxide

in the cement. However, excessive expansion was not confined to the specimens of No. 28039 but was likewise noted, although to a lesser degree, with the laminated calcareous shale No. 19374, and the opaline cherty shales Nos. 28045 and 28046 (Table 4, Appendix I).

The early tests in this series were made with 2.5%, 5%, and 10% additions to the Russian River sand. As evidence accumulated that excessive expansion occurred with even as low as $2\frac{1}{2}\%$ of No. 28039, the series was expanded to include specimens with lower percentages ranging down to as low as 0.10% (see Fig. 5(*a*)).

Other Tests

Later test series have included observations to determine the effect of:

(1) Age of cement;

(2) Washing soluble salts out of the cement, treating the aggregate with this resultant liquor, and then testing for expansion;

(3) Admixtures such as pumicite, celite, ground Ottawa sand, calcium chloride, and Vinsol resin;

(4) Treating the sand with sodium hydroxide and dilute hydrochloric acid;

(5) Varying the quantity of cement, using 1 : 3, 1 : 2, and 1 : 1 mortar mixes; and

(6) Methods of accelerating reaction by heating so as to reduce the time of test.

Most of these later tests have not been conducted long enough to produce conclusive data of value. Apparently, however, the test cannot be accelerated by heat, by the autoclave, or by pulverizing the deleterious materials, but can be accelerated by using 1 : 2 mortar mixes instead of 1 : 3 and adding from 5% to 20% of the reactive minerals crushed and screened to the size range, a −30-mesh to +80-mesh size.

The addition of a puzzolanic material seems to be effective, as 25% pumicite substituted for an equivalent weight of high-alkali cement reduced the expansion to a negligible amount at early periods. Part of the reduction may have been due to the lower percentage of total alkali in the combined mix. Ground Ottawa sand, however, was much less reactive and calcium chloride had little effect (see Fig. 7). The reaction with the puzzolanic-type cement included in Series II (*HP* in Fig. 4) was likewise considerably less than with other standard cements of the same or less alkali content. Vinsol resin appears effective to a certain extent.

Treating the Oro Fino sand with water mixed with, and then leached from, a high-alkali cement, or with a solution of sodium hydroxide, apparently stops any subsequent reaction; but treating with a hydrochloric acid solution is ineffective. This would indicate that it may be some form of silicate rather than carbonate which causes the trouble.

Concrete Tests

With the exception of Series I and IA, all of the tests described were made on mortar specimens. A later series consisted of 6-in. by 6-in. by 30-in. speci-

mens of six-sack concrete, using different combinations of good and bad coarse and fine aggregates. Specimens of each mix were cured, sealed in water, and were kept continuously dry at laboratory humidity and temperature. This test has been in progress too short a time, however, to produce information of value.

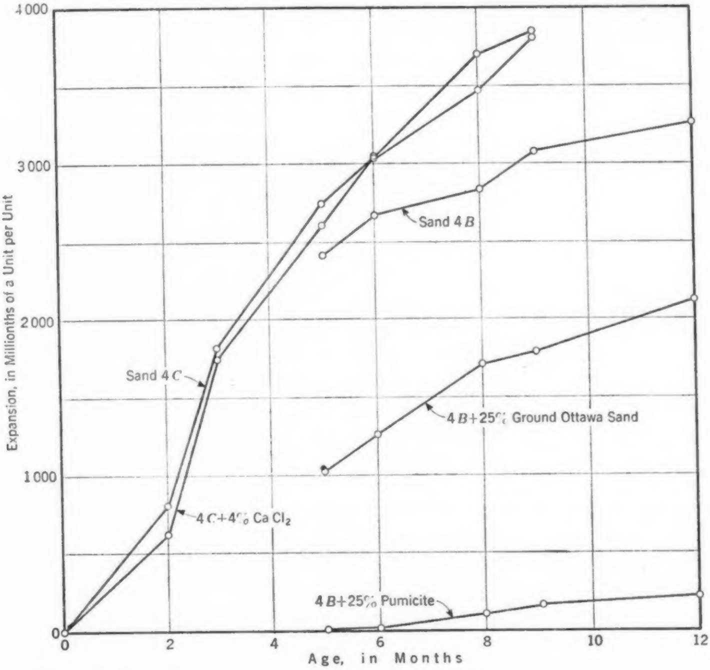


FIG. 7.—EFFECT OF VARIOUS ADMIXTURES ON THE EXPANSION OF A HIGH-ALKALI CEMENT AND REACTIVE SAND MORTAR (1 : 3 MIX)

SHALE AND CHERT POPOUTS

Although the laboratory investigations described herein conclusively demonstrate a very destructive chemical reaction between a high-alkali cement and some ingredient in the siliceous magnesian limestones, nevertheless one cannot overlook the fact that some chemical action is also taking place in the presence of some types of the low lime-magnesian shales and cherts, as evidenced by the data shown in Table 1, as well as by a study of the extensive popouts observed in numerous structures, including pavements, bridges, and buildings where shale or chert particles are invariably found at the base of the popouts, frequently accompanied by a soft viscous substance analyzed as a silica gel with some form of sodium silicate. Figs. 8(*a*) and 8(*b*) are views of large popouts from the basement of the old Times Building in Los Angeles. Fig. 8(*a*) is a part of a 5-in. popout showing at: (*a*) Sodium silicate gel; (*b*) an intermediate zone of partly altered shale; and (*c*) apparently unaltered rock. The chemical analyses

of these samples, given in Table 2, were necessarily made on very small specimens and may therefore be inaccurate in some details. Fig. 8(*b*) shows altered and unaltered parts of the mineral that caused the popout.

The aforementioned silica gel frequently has been observed exuding from cracks or pores in the face of concrete structures in which aggregates and ce-

TABLE 2.—CHEMICAL COMPOSITION OF ROCK AND GEL FOUND IN THE TWO TYPICAL POPOUTS SHOWN IN FIGS. 8(*a*) AND 8(*b*)

Description	Formula	Fig. 8(*a*): Gel	Fig. 8(*a*): Intermediate altered shale	Fig. 8(*a*): Apparently Unaltered Rock: Upper layer	Fig. 8(*a*): Apparently Unaltered Rock: Lower layer	Fig. 8(*b*): Gel	Fig. 8(*b*): Apparently unaltered rock
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Moisture	....	12.63	8.92	3.12	3.23	9.69	3.38
Combined H_2O and organic matter	....	9.81	5.84	2.80	3.41	6.96	5.00
Iron	Fe_2O_3	2.40	1.92	3.83	3.83	3.19	2.55
Aluminum	Al_2O_3	4.38	5.84	1.09	6.37	4.61	4.69
Lime	Ca O	2.90	2.36	0.92	2.50	2.62	1.36
Magnesia	Mg O	0.58	0.72	0.54	1.09	0.85	0.79
Alkalis	Na_2O	12.93	9.62	2.04	4.90	12.77	5.00
Carbon dioxide	CO_2	....	2.50	0.71	1.43	3.93	1.79
Chlorides	Cl	....	0.72	0.40	0.24	0.48	0.16
Silica	Si O_2	53.86	60.16	83.30	71.22	53.40	74.00
Undetermined	....	0.51	1.40	1.25	1.78	1.50	1.28
Total	....	100.00	100.00	100.00	100.00	100.00	100.00

ments similar to those under investigation had been used. This type of gel has also been found throughout the cores and pieces of concrete examined from all of the failed pavements and structures. In some cases a number of aggregate particles had entirely altered to gel, so that it often swelled out above a freshly cut surface of a concrete specimen. In many cases the reaction has only partly progressed into particles identified as siliceous shales or cherts of a type relatively free from lime and magnesium carbonates. Although the gel has usually been found in a plastic state in fresh popouts, and in specimens cut from beyond the exposed surfaces of the concrete pavements and structures, it hardens readily and shrinks when dried on exposure to the air.

A detailed examination was made of approximately 200 popout and core specimens removed from the affected buildings, sea walls, bridges, and pavements over an area extending from Los Angeles on the south to the northern part of the Salinas Valley. As defined by Allan H. Nicol, mineral technologist for the State Division of Highways, each popout particle had associated with it a considerable amount of optically isotropic material of very low index of refraction. This material was identified as opaline silica with complex sodium silicates and was found to have indexes ranging from 1.41 to 1.54. This substance, which was slightly plastic before drying, is hereafter referred to as "gel." It was found that the index of refraction of the dried gel increased with the color. The clear, glassy variety had indexes ranging from 1.41 to 1.47.

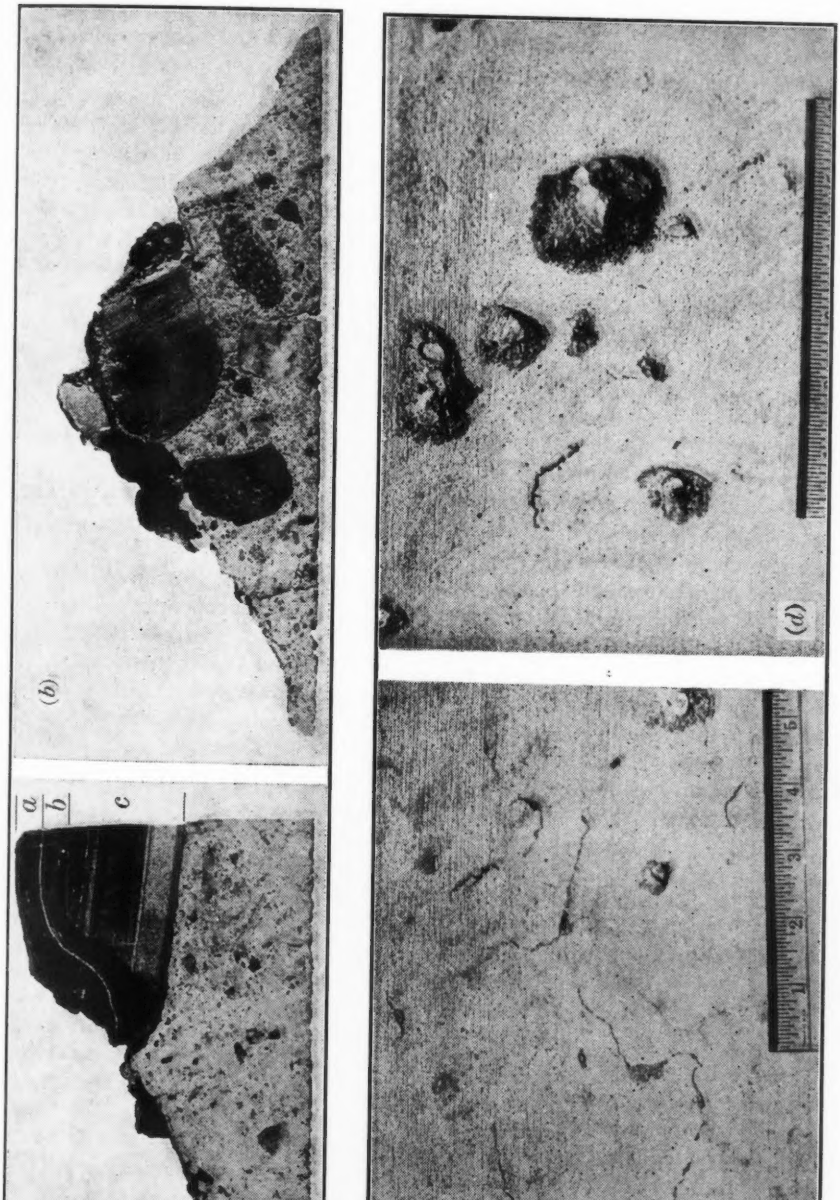


FIG. 8.—TYPICAL SHALE POPOUTS

The yellow and pale brown types showed values from 1.46 to 1.49, whereas with the brown to dark brown types the values ranged from 1.47 to 1.54.

Microchemical tests showed that the type of gel with a low refractive index was a nearly pure opaline silica, relatively free from sodium, magnesium, aluminum, calcium, and iron salts, whereas the yellow, amber, and brown varieties, which predominated in many specimens, contained increasing percentages of silicates and other compounds of these and other elements. The presence of these compounds, together with a variable content of adsorbed water, undoubtedly could account for the variable refractive indexes.

Most of this gel is fluorescent under ultraviolet light. A strong fluorescence was observed in the expansion bars and mortar cylinders which carried various percentages of No. 28039 (siliceous magnesian limestone; see Appendix I). These specimens had been subjected to the sealed curing test. The fluorescence in this case is due to gel derived from the particles of No. 28039 in the aggregate, as the rock in its original unaltered condition does not fluoresce.

The gel-like material associated with these specimens contains opaline silica, with varying amounts of sodium, magnesium, aluminum, calcium, and iron compounds. All of the gel examined on popouts, and in the cores and broken concrete from the various sources, shows similarity of optical properties and chemical composition.

Siliceous, calcareous, and cherty shales were the principal petrographic types associated with the popouts and the gels found in the concrete specimens. Chert was also commonly found, but it was impossible to determine how many of the completely altered particles may have been chert before the reaction.

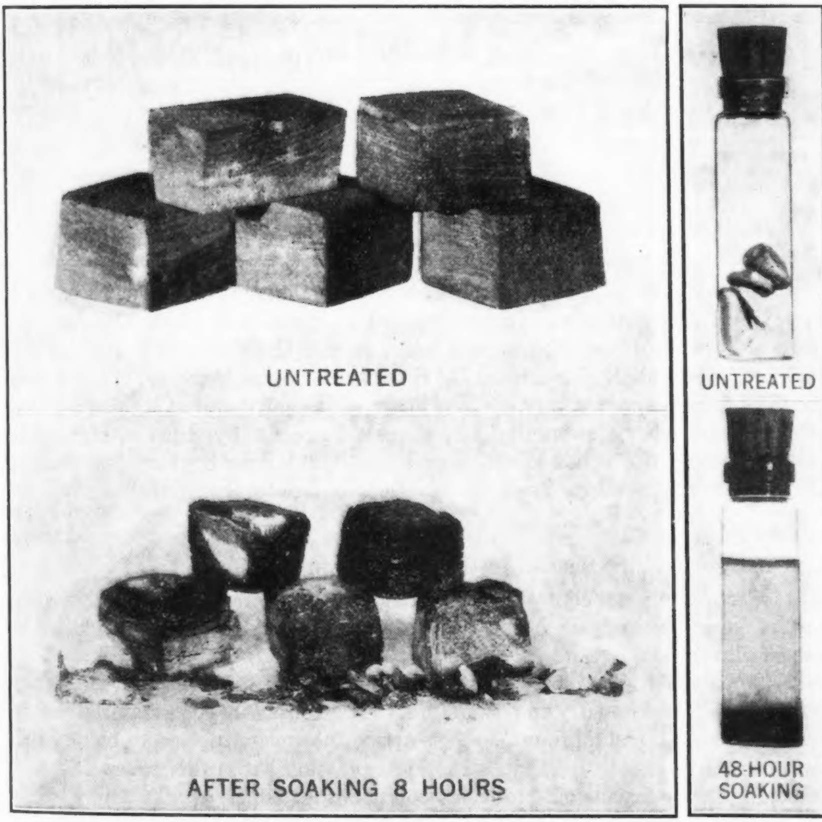
The dark gel carrying the impurities was usually associated with a dark-colored, organic, siliceous shale or chert. The light-colored gel was normally found with light-colored shale or chert, usually cream, tan, or buff. These light-colored shales and cherts were generally more free from organic material and fossil remnants than their dark-colored equivalents.

It can be reasoned that this substance forms in the presence of moisture through the action of alkali in the cement or possibly in the rock on some form of silica in the rock, resulting in a gel-like end product consisting primarily of opaline silica, sodium silicates, and water. As a partial proof of this assumption, white deposits or incrustations containing more than 50% Na_2CO_3 have been found on the protected parts of structures where neither surface water nor ground water could come directly in contact with the concrete surface on which the deposit occurred. Presumably this sodium was in the form of sodium hydroxide in the concrete and was changed to sodium carbonate by carbon dioxide derived from the air.

Following this reasoning, mineralogy has well established the fact that hydrous forms of silica, such as opaline, are readily dissolved in either sodium or potassium hydroxide solutions, and also that these forms of silica are found in abundance throughout some of the Miocene formations in the area under discussion. All siliceous shales and cherts, therefore, are subject to question, in connection with the chemical reaction resulting in the gel substance. Extensive tests show that many types of shale and chert found in the gravel deposits are readily dissolved and disintegrated by soaking in a 10% solution of sodium

hydroxide. In Fig. 9 the effects of soaking are shown for two materials—specimens cut from ledge rock, and ½-in. shale and chert particles from stream-bed gravel deposits.

Although these tests show only that the gel type of chemical action accompanying popouts and expansion failures can be approximately duplicated in the laboratory by the use of a sodium hydroxide (Na OH) solution, additional tests indicate that a similar action occurs when these types of aggregate are soaked in the mixing water from high-alkali cements. The chemical composi-



(*a*) Specimens Cut from Ledge Rock (*b*) Stream-Bed Gravel

FIG. 9.—EFFECT OF A 10% SODIUM HYDROXIDE SOLUTION (AT 70° F) ON SOME TYPES OF SILICEOUS (CHERTY) SHALE AND CHERT FROM THE CALIFORNIA MIOCENE FORMATIONS

tion of water containing alkali, leached from a low-alkali (0.45%) and a high-alkali (1.14%) cement, is shown in Table 3, together with the change in each solution, after digesting for 120 hr at 200° F with Oro Fino sand containing about 9% of chert and shale particles. It is significant that the water from the

high-alkali cement dissolved approximately ten times more silica during the treatment than the water from the low-alkali cement.

Figs. 8(*c*) and 8(*d*) show two typical popouts of the type accompanied by relatively large deposits of gel. It will be noted from Table 2 that these shale particles have chemical compositions somewhat similar to No. 28042 (Table 4, Appendix I), particularly in regard to lime, magnesia, and alkali, although the shale in Figs. 8(*a*) and 8(*b*) is very much harder than that in Figs. 8(*c*) and 8(*d*), or the soft shale represented by No. 28042. Even if the gel type of reaction accompanies many popouts and other expansion failures observed in the field and also can be accounted for, at least theoretically, when a high-alkali cement is used, the volume change and the exact source of the stress causing the popouts and accompanying the reaction require further investigation.

TABLE 3.—CHEMICAL COMPOSITION OF WASH WATERS FROM A LOW-ALKALI AND A HIGH-ALKALI CEMENT AND THE SAME SOLUTIONS AFTER DIGESTING FOR 120 HR AT 200° F WITH ORO FINO SAND CONTAINING ABOUT 9% OF SHALE AND CHERT PARTICLES
(In Parts per Million)

Description	Cl	CO_3	$H\,CO_3$	OH	SO_3	$Si\,O_2$	Al_2O_3 (Fe_2O_3)	Ca O	Mg O	Na_2O[a]
WASH WATER FROM:										
Low-alkali cement........	20	75	....	1,400	10	7	....	450	....	2,200
High-alkali cement........	60	240	....	3,200	36	39	....	110	....	6,000
SOLUTIONS AFTER DIGESTING WITH SAND:[b]										
Low-alkali cement........	4	250	....	870	23	590	42	....	....	1,800
High-alkali cement........	50	660	....	2,800	45	6,100	....	....	....	5,600

[a] Alkalies (Na and K) reported as Na_2O. [b] 3-mesh to 10-mesh sand particles.

CONCLUSION

The tests described herein justify the following conclusions:

1. Certain mineral constituents in concrete aggregates contribute to expansion of concrete and sometimes develop stresses of such magnitude as to cause failure.

2. Some shales expand excessively when saturated with water or when they are alternately wet and dry and, therefore, the percentage of such material should be kept to a practical minimum. This action, however, appears to be physical and of much less intensity than a chemical reaction with other minerals.

3. Excessive expansion, sufficient to rupture a concrete mass, may occur when certain minerals are present. The reaction in this case is chemical, and evidence indicates that it always takes place with the siliceous magnesian lime rocks found in the aggregates from the Upper Miocene sedimentary deposits of the state and frequently in the presence of some of the low-magnesia, low-lime shales and cherts.

4. The chemical reaction producing excessive expansion apparently occurs only when the portland cement component contains an appreciable percentage of alkali in the form of sodium and potassium oxides. It is of an intensity proportional to the percentage of such oxides, apparently being of such low order as to be negligible when the alkali content is less than 0.6%.

Acknowledgments

The investigations and tests described in this paper were conducted as a research project of the California Division of Highways, of which State Highway Engineer Charles H. Purcell, Assoc. M. Am. Soc. C. E., is chief. The writer is indebted to the following members of his staff, all of whom have contributed valuable suggestions: Lester Meder, assistant physical testing engineer, for having first observed the accelerated expansion of specimens cured in sealed containers and for subsequent direction of the fabrication and measurement of test specimens and intelligent analysis of results; G. H. P. Lichthardt, chief chemist, for the direction of all chemical analyses and the development of the theory of the probable reactions; O. J. Porter, senior physical testing engineer and head of the Aggregate and Soils Department, and his assistant, Mr. Nicol, for their contribution to the geological and aggregate classification and discussion, identification of the mineral constituents, and suggestions as to probable reactions. In the field District Engineers L. H. Gibson, District V, San Luis Obispo, and S. V. Cortelyou, M. Am. Soc. C. E., District VII, Los Angeles, afforded wholehearted cooperation.

APPENDIX I

Mineral Identification

The chemical analysis and physical properties of a few of the Miocene shales, cherts, and impure limestones, found in the coast area between Monterey and Los Angeles, Calif., are given in Table 4. Each sample is identified by a laboratory test number that identifies the following detailed description of each one:

No. 19374—Laminated Calcareous Shale:

Groundmass an impalpable mixture of clay, opaline silica, and sericite, carrying abundant fragments of quartz, orthoclase, chalcedony, and calcite, with some marcasite. Organic matter and fossil relics, including foraminifera and diatoms, are found occasionally as stringy accumulations parallel with the laminæ. The chalcedony occurs chiefly as fragmental grains, with an occasional veinlet. A few small clay lenses were observed without any appreciable embedded mineral grains of sufficient size to be identified. The laminations in this rock are quite high in clay and opaline silica, and carry the thickest layers of organic matter and fossils. This could explain the ease with which this material splits apart.

No. 19375—(Chalcedonic) Cherty Shale:

Predominant evidence of chalcedonic replacement of the shaly portions of this rock. The groundmass is predominantly chalcedony of the flamboyant type, with some areas not entirely replaced. Sizable fragments of recognizable minerals are lacking. Organic matter, low in content, occurs in layers that tend to parallel the bedding. The entire specimen is pierced throughout by chalcedony veinlets. Quartz and feldspar fragments are lacking. Opaline silica not observed in this material. Fossils absent.

TABLE 4.—CHEMICAL ANALYSIS AND PHYSICAL PROPERTIES OF SHALES, CHERTS, AND LIMESTONES
(All Values Are Percentages)

No.	Description	Formula	LABORATORY TEST NOS.: 19374	19375	28037	28038
1	Moisture	....	0.75	0.22	3.88	2.14
2	Organic matter[a]	....	1.42	0.98	2.21	2.63
3	Iron	Fe_2O_3	3.38	2.64	2.59	2.59
4	Alumina	Al_2O_3	1.40	3.15	13.11	0.79
5	Lime	Ca O	6.01	0.65	2.71	Trace
6	Magnesia	Mg O	0.60	0.51	1.69	Trace
7	Alkali	Na_2O	0.58	0.78	2.11	1.43
8	Manganese	Mn_2O_3	0.01	0.03	0.36	....
9	Carbonates	CO_2	9.46	0.68	2.31	....
10	Undetermined	....	0.69	0.14	0.61	0.48
11	Silica	Si O_2	75.70	90.22	68.42	89.94
12	Totals	....	100.00	100.00	100.00	100.00
13	Average loss, Na_2SO_4 (percentages)	....	22.0	....	36.4	28.7
	Rattler Test:[b]					
14	100 revolutions	....	5.2	....	8.0	9.4
15	500 revolutions	....	23.0	....	31.2	34.6
	Specific Gravity:					
16	Le Chatlier	....	....	....	2.20	2.00
17	Saturated surface dry	....	1.82	....	1.80	1.89
	Absorption:					
18	Percentages	....	8.0	....	10.2	4.3

TABLE 4—(*Continued*)

No.	28039	28039A	28042	28045	28046	30704
1	0.42	0.92	3.92	4.74	5.09	3.8
2	4.15	0.58	3.12	Trace	2.19	8.3
3	0.83	0.99	3.24	3.19	3.52	1.1
4	0.27	1.22	12.40	5.09	3.90	3.5
5	26.24	30.85	1.80	5.18	3.58	0.4
6	15.55	8.75	1.33	0.52	0.52	Trace
7	0.20	0.53	4.21	2.09	1.87	....
8	0.02	....	....	....	....	....
9	33.92	37.03	0.44	2.50	2.30	....
10	0.58	0.39	0.84	0.44	0.66	1.6
11	17.82	18.74	68.70	76.25	76.37	85.1
12	100.00	100.00	100.00	100.00	100.00	100.00
13	2.8	....	72.7	56.3	48.2	....
14	4.8	....	11.2	10.2	....	....
15	20.0	....	41.8	36.0	....	....
16	2.62	....	2.50	2.28	2.16	....
17	2.43	....	1.90	1.94	1.94	....
18	1.7	....	7.1	8.4	7.7	....

[a] Combined water and organic matter. [b] Los Angeles Rattler Test.

No. 28037—Diatomaceous Shale (Soft Siliceous Type):

Carries a few fragments of plagioclase and quartz. Groundmass consists chiefly of diatoms and other opaline material, together with clay, some mica (probably sericite), and a variable amount of organic matter. Groundmass is quite uniform.

No. 28038—Chert:

Opaline material, with some organic matter, and a few fine chalcedony fragments constitute the groundmass of this speciman. The groundmass is pierced throughout the entire length of the slide by a series of fine chalcedony veinlets that average about 0.15 mm in width. These veinlets

are all more or less parallel with each other and are apparently conformable with the bedding. They also parallel the heavy layers of organic matter. Abundant evidence of chalcedonic replacement throughout groundmass. "Canals" or veinlets are in portions partly opaline. No fossils or diatoms were noticed. Chalcedony of veinlets is of flamboyant type and carries minute opaque inclusions, possibly marcasite. Many canals or veinlets not entirely sealed from wall to wall by opaline cement, leaving innumerable fissures and openings.

No. 28039—Siliceous Magnesian Limestone:

A groundmass composed principally of chalcedony, opal, dolomite, and calcite. The dolomite occurs as rhombohedrons; calcite shows excellent twinning lamellæ; the chalcedony is flamboyant and shows aggregate polarization under *X* nicols. A few diatoms present in the groundmass, many showing replacement by calcite, dolomite, or chalcedony. Irregular patches of organic matter, concentrated near the chalcedony.

No. 28042—Soft Shale (Santa Margarita Formation):

Quartz and other mineral grains are very abundant in this material (over 25% of groundmass). Groundmass carries abundant organic matter and finely disseminated marcasite. No evidence of siliceous replacement. No diatoms or other micro-fossils observed. Cracks and fissures numerous. Cement of groundmass chiefly clay. No evidence of recrystallization—hence, material is probably quite soft and friable. Average size of grains approximately 0.039 mm. Consist principally of quartz, with some plagioclase and actinolite. Quartz has inclusions, possibly rutile. Organic matter strung out parallel to laminæ. This sample also carries a considerable amount of an anisotropic, pleochroic mineral (index greater than balsam) found in popouts.

No. 28045—Opaline Cherty Shale:

This rock shows strong similarity to No. 19374 and to No. 28046, under low-power magnification. It is of the same geological age as No. 19374 and No. 28046, as evidenced by the foraminifera and other fossils, which show partial chalcedonic replacement. The groundmass appears to be an intimate mixture of chiefly opaline silica with a small amount of diagenetic glauconite and clayey portions, with traces of organic matter. The latter is of fairly uniform distribution throughout the slide. Embedded in this groundmass are abundant coarse detrital grains of quartz, orthoclase, and plagioclase. Numerous feldspars appear partly glauconitized. Chalcedony is not very abundant in the groundmass, except as a replacement of the fossils. No free calcite or dolomite as fragments or veins was observed. This material differs from No. 19374 by a lower calcite-dolomite content, and by a higher amount of detrital quartz and feldspars, as well as a more uniform distribution of the organic matter.

No. 28046—Opaline Cherty Shale:

This rock is of the same geological age as No. 19374 and No. 28045 and strongly resembles them. The following differences, however, are noteworthy: In No. 28046 the detrital quartz and feldspar content is lower, and these grains are smaller; spherules of chalcedony are fairly abundant in the groundmass; glauconite and organic matter are more plentiful than in No. 28045 and occur in concentrated form parallel with the laminæ; although the groundmass is composed chiefly of opaline silica, it nevertheless appears partly pleochroic under high power and indicates the presence of intermixed diagenetic minerals (formed after deposition). No free dolomite or calcite was observed in this material, and what lime content is found on chemical analysis will be due in large part to the fossils and the

detrital plagioclase grains. Due to concentration of organic matter and clay in definite layers, this rock is more laminated than No. 28045, and the evident lack of opaline replacement along these laminæ suggests the easy cleavability of this material.

No. 30704—(Dark) Opaline Chert:

Dark brown stringy accumulations of organic matter parallel the bedding, which is frequently distorted. An occasional quartz or orthoclase fragment is enclosed by opaline groundmass. Sample carries a few channels of brecciated material, cemented by opaline veinlets and clay. The quartz and orthoclase are the only recognizable anisotropic minerals observed in this material. Matrix predominantly opaline silica with some undistinguishable clay minerals. Material shows no evidence of chalcedonic replacements, and no fossils.

APPENDIX II

Sand Identification

Group I.—In this group rock fragments consist chiefly of granitic rock and sandstone with less than 15% quartzite, rhyolite, andesite, siltstone, and limestone, and from 4% to 15% shale and chert. The mineral grains consist chiefly of quartz and feldspar, with less than 10% of pyroxene, amphibole, mica, magnetite, hematite, limonite, serpentine, and calcite. Sands Nos. 1 to 6 in this group were secured from the following places:

Sand No.	Source
1	Santa Clara River at Saticoy, Ventura County
2	Sisquoc River at Sisquoc, Santa Barbara County
3	Piru Creek at Santa Clara River, Ventura County
4	Oro Fino, tributary to Salinas River, Monterey County
5	Bank adjacent to Salinas River at Atascadero, San Luis Obispo County
6	Salinas River at Templeton, San Luis Obispo County.

Group II.—Rock fragments in Group II consist chiefly of granitic rock and gneiss in Southern California, and of graywacke (sandstone) in Northern California, with less than 10% of basalt, andesite, quartzite, and jasper and usually less than 1% of shale and chert. Mineral grains in Group II are similar to Group I. Sands Nos. 20 to 24 in this group were found in:

Sand No.	Source
20	Coyote Creek, Santa Clara County
21	Russian River, Healdsburg, Sonoma County
22	Livermore Valley, Alameda County
23	Near Olympia, Santa Cruz County
24	Roscoe, San Fernando Valley, Los Angeles County
25	Irwindale, San Gabriel Valley, Los Angeles County
26	Monrovia, San Gabriel Valley, Los Angeles County
27	San Joaquin River at Friant, Fresno County

Group III.—The sands in Group III were made by crushing ledge rock to the desired gradation. Sands Nos. 101 and 102 in this group were found in:

Sand No.	Source
101	Crushed hornblende diorite, Logan, San Benito County
102	Crushed quartz, Jackson, Calaveras County.

APPENDIX III

Cement Classification

The cements used in the investigation were analyzed as shown in Table 5.

TABLE 5.—Cement Analyses

Cement identification	Oxide Analysis (Percentages)							Compound Composition (Percentages)				Merriman Alkali Test		Autoclave test (percentage experiment)
	SiO_2	Fe_2O_3	Al_2O_3	CaO	MgO	SO_3	Alkali as Na_2O	C_4AF	C_3A	C_3S	C_2S	Alkalinity	Free alkali	
AS	22.76	2.00	4.02	66.04	1.80	1.54	0.45	6.1	7.2	62.3	17.9	3.3	1.0	0.05
BL	23.76	2.48	4.34	64.58	1.61	1.61	0.51	7.6	7.2	41.1	37.2	4.1	4.1	0.12
CL	20.42	5.68	5.92	62.86	1.57	1.74	0.77	17.3	5.8	52.5	17.6	3.6	2.5	0.09
DL	22.26	3.60	4.73	63.18	2.93	1.38	0.90	10.9	6.4	48.5	26.8	4.1	5.1	0.21
EL	21.57	3.52	4.49	62.44	4.08	1.50	0.92	10.6	6.1	50.8	23.3	3.8	3.7	0.25
FS	21.00	3.12	5.39	64.80	1.84	1.51	0.92	9.4	9.0	61.0	13.5	3.8	3.6	0.10
FL	23.34	3.96	4.28	64.32	1.09	0.87	0.50	12.1	4.6	49.5	28.8	3.5	2.6	0.05
FA	25.35	3.87	3.42	63.52	1.62	1.22	0.56	11.8	2.6	34.6	46.2	3.6	2.0	0.01
GS	21.48	2.72	5.64	64.50	1.72	1.71	1.14	8.2	10.3	54.1	20.3	5.1	6.3	0.09
GL	20.02	5.35	5.49	62.80	1.65	1.81	1.10	16.4	5.6	58.2	12.0	5.0	6.2	0.05
HS	21.14	2.48	6.43	64.16	2.23	1.78	0.69	7.5	12.8	49.5	23.0	3.3	1.5	0.18
HL	21.59	4.80	5.17	62.61	1.72	1.89	0.53	14.6	5.6	45.2	27.3	3.5	1.4	0.05
HP	31.10	2.40	6.77	53.30	1.86	1.57	0.78	Portland puzzolanic type				....	....	0.01
IS	22.56	2.40	5.52	65.33	1.48	1.49	0.44	7.3	10.6	50.1	26.7	3.2	2.1	0.17
M	21.30	2.90	5.70	63.44	2.78	1.97	0.83	8.8	10.3	49.4	23.4	4.2	4.4	0.12
N	22.34	3.78	4.78	63.32	3.44	1.51	0.47	11.5	6.3	46.1	29.3	3.1	1.6	0.10
O	21.65	3.55	5.65	63.35	1.45	1.85	0.51	10.8	9.0	46.0	27.0	3.7	3.9	0.01
R	22.37	4.00	4.35	64.65	1.36	1.45	0.64	12.2	4.8	54.5	22.8	3.8	2.7	0.01

In the identifying letters given in the first column, the first letter refers to the brand and the second letter to the type of cement, as follows:

S = Standard cement; in some cases the standard product contains less than 8% of C_3A and therefore might also be classified as a low or moderately low C_3A cement.

L = Cement having less than 8% C_3A and complying with American Association of State Highway Officials Specification M–60–38 as distinguished from the standard product of the same manufacturer with higher C_3A content.

A = Low-heat type "*A*."

P = Portland-puzzolan; insoluble, 15.44; all other cements less than 0.85% insoluble; cements *M*, *N*, *O*, and *R* are from mills outside California.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

BRIDGE AND TUNNEL APPROACHES

Discussion

By John F. Curtin, Jun. Am. Soc. C. E.

John F. Curtin,[17] Jun. Am. Soc. C. E. (by letter).[17a]—The primary premise of this paper is that the approaches to bridge and tunnels govern the capacity and accessibility of these structures. Corollary with this is the premise that the characteristics of the motor vehicle and the inherent limitations of its operator are primary considerations in the design of these facilities.

It is gratifying to note that all of those who have been kind enough to submit discussions on this paper are substantially in agreement with the main contentions. Some of them have contributed additional material to substantiate the premises and to broaden their application. The writer is grateful for these additions as well as for the discussion in which exceptions are made to certain of the design considerations as presented. The latter is more interesting, perhaps, for it reveals the statements wherein the writer has not been entirely clear as well as those upon which a difference of opinion exists. Some further comment on these points is warranted in order to correct the misunderstandings and to clarify the disagreements.

As Mr. Edwards states, the obsoleteness of the approaches to the Brooklyn and Queensboro bridges in New York City, although their main spans are still useful, is evidence of the need for greater consideration upon entrance and exit facilities. The City of New York is now (1940) spending $68,000,000 for the Queens Midtown Tunnel, and plans to spend another $80,000,000 for the Battery Tunnel in order to handle the increased volume of traffic crossing the East River. It is doubtful that either of these tunnels would be necessary if the approaches of the Brooklyn and Queensboro bridges could be developed to their full potential capacities. Both of the new facilities will tap the same traffic sheds that these two bridges, together with the Manhattan and Williamsburgh bridges, are serving. However, the two new tunnels will provide probably a more economical, and certainly a more effective, solution to the East

Note.—This paper by John F. Curtin, Jun. Am. Soc. C. E., was published in November, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1940, by Messrs. Dean G. Edwards, and John W. Beretta; March, 1940, by George Hartley, Esq.; and April, 1940, by Messrs. George H. Herrold, Park H. Martin, and Charles M. Noble.

[17] Highway Engr., Economist, Am. Petroleum Inst., New York, N. Y.

[17a] Received by the Secretary October 28, 1940.

River crossing problem, because the cost of the major operation which would be necessary to modernize the approaches to the older bridges now would be prohibitive.

In a city such as New York, the coordinated movement of traffic is so dependent upon the arterial plan of the city that it frequently is more economical, in serving an area, to build a group of facilities with less extensive approaches than to attempt to converge all traffic upon one or two crossings. As Mr. Herrold suggests, the comprehensive city plan may dictate the necessity of having several facilities spaced some distance apart.

For a large city, the problem resolves into one of determining the number of crossings which, in conformity to the city plan, will afford the maximum convenience in traffic movement with a minimum of backtracking and overlapping, at the same time considering the relationship between the cost of these traffic facilities and the transportation benefits that are derived from them. When this ground work is completed, the approaches of each bridge or tunnel can then be designed so that the crossing will fit into the arterial plan and handle its proportionate share of traffic with maximum convenience.

This arrangement should prevail regardless of how the structure is financed. Mr. Noble states it aptly in his conclusion that the various traffic studies together with the cost estimates and a recognition of the community aspects "constitute the only intelligent method of determining the specific location and the general design of the approach system." For this reason, the writer cannot agree with Mr. Martin's contention that the location of a non-toll project might differ materially from that of a structure to be financed by tolls. Although it is true that topography, land value, and other physical considerations must be weighed in selecting the site of a crossing, fundamentally it is the fitness of the project as a traffic facility, not its cheapness, which determines its value to the motoring public and to the community.

Only too often engineers have been misguided, by shortsighted economy, into selecting a cheaper site or designing an inadequate approach for a structure. This is particularly true on the so-called "free" projects, where the design does not have to stand the "cash register test." Regardless of whether the motorist pays for the crossings directly through tolls or indirectly through gasoline taxes, it remains the responsibility of the engineer to determine the location of the structure and the design of its approach system that will accommodate the coordinated movement of traffic in an economic manner—without compromising that objective.

The reason that the writer has dealt mainly with toll structures is, as Mr. Herrold states, that "more attention has been given to approaches where tolls are charged than to approaches to tunnels and bridges which have been paid for through taxes." Although the economic considerations have been treated only incidentally in this paper, due to limitations of space and more adequate treatment by other writers, it is evident from inspection of them that the approaches of toll structures are superior economically as well as functionally to those of non-toll projects. The self-liquidating facilities have a better ratio of total motoring benefits to total costs since they are scrutinized more carefully in the planning stage from that angle in order to insure solvency. The implica-

tion by one writer that federal "largess" has obviated economic considerations with respect to toll projects is totally unfounded.

However, there is no real reason why the same principles of economic analysis cannot be applied to non-toll projects. Heretofore, engineers have relied mainly on a comparison of cost estimates in determining which in a group of approach studies for a bridge or tunnel is most economical. The various items of expense, property, construction, maintenance, and operation are estimated for each plan, and since the total of these is the most tangible sum that can be assigned to that proposal, it becomes the basis of comparison. Nevertheless, the inadequacy of comparing only the total costs is well recognized, for the final selection is generally made by applying a judgment factor, which embraces an understanding of the other considerations, to the comparative costs. Furthermore, although most bridges and tunnels indicate that the final selection of site and approach development was the most satisfactory solution, there are many which show that one or more of the intangible considerations were slighted or neglected.

If the full return is to be had upon the investment in a bridge or tunnel, a more inclusive analysis must be used in determining the proper location and development of the approaches. As Mr. Martin states, "the final decision * * * must be based on the functional adequacy of the device, with due consideration to the cost." Although it is more difficult to prepare than the cost estimate, an analysis of the potential traffic and the benefits that will accrue to it by use of the proposed facility should also be made in terms of monetary value. Nathan Cherniack, Assoc. M. Am. Soc. C. E., has developed an effective method for measuring the potential traffic of a proposed vehicular crossing in terms of transportation benefits which he described in an excellent paper.[18] In addition, John Beakey and C. B. McCullough, M. Am. Soc. C. E., have derived a set of monetary relationships among the cost of a highway facility, its potential traffic, and the benefits which it will provide for that traffic.[19] By considering all items on the common denominator of dollars and cents, the correlation between factors may be seen prosaically and a concrete index of the value of each proposal may be had. That site and its approach development which has the highest ratio of benefits to costs is the most desirable from the standpoint of the motoring public, whether a toll project or not. For a toll structure, the plan offering the greatest over-all benefits will attract the most income to the facility; for a non-toll structure, it will serve the public best. Although tolls are an important financial consideration, they are only a means of financing and, therefore, bear no direct relationship to the physical planning other than to require certain provisions for collecting them.

The conclusion drawn by Mr. Beretta that only the approaches of large bridges and tunnels have received proper attention in design is generally true, although there are some excellent examples of approach development on smaller structures. As a rule, the traffic design problems on a small vehicular crossing

[18] "Measuring the Potential Traffic of a Proposed Vehicular Crossing," by N. Cherniack, *Proceedings*, Am. Soc. C. E., February, 1940, p. 207.

[19] "Economics of Highway Planning," by John Beakey and C. B. McCullough, *Technical Bulletin No. 7*, Oregon State Highway Commission, 1937.

are not as complex as those on a large bridge or tunnel and, therefore, the approaches should be more adaptable to the ideal solution. However, the relative size of these crossings and the fact that frequently the design of such structures is standardized in many respects tend to lessen their importance as traffic facilities so that their approaches are more likely to be neglected. Messrs. Beretta and Herrold concur in the opinion that small river crossings could be improved materially and extended in service life, if more consideration were given to these structures from the standpoint of traffic design.

Apparently, some difference of opinion exists as to the usefulness of circles for regulating traffic on bridge and tunnel approaches in cities. Mr. Martin states that they have been economical and satisfactory in his experience, whereas Mr. Edwards demonstrates their ineffectiveness in handling traffic in Washington, D. C. The writer is inclined to agree more with the conclusions of the latter—that traffic circles should be limited to rural and suburban regions.

Because of their successful adaptation to the intersections of heavily traveled highways, the installation of circles as approaches became popular in the late 1920's. In quick succession, the planning engineers of the Goethals Bridge, Outerbridge Crossing, and the Delaware River Bridge adopted the rotary principle for one approach of each of these structures. Its advantage over the street extension and reservoir types of approaches was the presumption of continuously moving traffic and the elimination of the bothersome left turn. By compelling all vehicles to proceed in one direction around a circle, the approach became capable of handling a larger volume of traffic with less delay. The theory of operation is that conflict is minimized by the substitution of a continuous weaving operation by vehicles moving in the same circular direction for a direct crossing. This principle is discussed fully by Guy Kelcey, M. Am. Soc. C. E., in his excellent paper.[20]

The size of the central island has a considerable influence on the efficiency of a traffic circle because the radius of the roadway must be large enough to accommodate a reasonable speed, and sufficient distance must be provided between connections to the circle for the interweaving of vehicles. This latter factor tends to slacken the speed and reduce the capacity because of the gaps created by cars weaving in and out on the circular roadway. The necessity of moving across the circle causes a greater heading between vehicles and consequently reduces the lane capacity. In addition, traffic entering and leaving the crossing frequently must merge on one or two quadrants of the circular roadway. This induces additional interference and further reduces the effective capacity of each lane. Furthermore, the circling operation often causes motorists to become confused and to turn off at the wrong roadway.

Rotaries are being used with moderate success in undeveloped areas; in cities, however, the waste of land in the central island makes them impractical. This island cannot be occupied without seriously impairing the operation of traffic and may be used only for decorative statues or landscaping. Pedestrian access creates a dangerous accident hazard and vehicular access

[20] "Channelization of Motor Traffic," by Guy Kelcey, *Proceedings*, Am. Soc. C. E., December, 1939, p. 1649.

invites operational interference as well as the loss of one lane due to parking. Furthermore, as Mr. Edwards states, circles operate ineffectively under heavy traffic and do not obviate the use of lights.

Virtually the same criticisms may be made of the reservoir approach, for on this type, the unwieldy maneuvering between the rectangular plaza and the roadway of the bridge or tunnel reduces the effectiveness of the facility unless a tapered area is provided between them, in which case, the rectangular space is largely unnecessary. Furthermore, the rectangular area is not fully used, because it is ill-adapted to vehicular movement; as much as 40% of it is wasted as far as storage space is concerned. This is illustrated at the Holland Tunnel Approach, Fig. 16, to which the paper and Mr. Martin's comment specifically referred. Unlike a parking lot, the storage space of an approach is used only momentarily for that purpose, and therefore it should be shaped so as to accommodate the continuously converging operation of vehicles.

Notwithstanding the comment by Mr. Martin that circles are especially adaptable to flat areas, it has been the observation of the writer that the most effective traffic circles are those in which the third dimension, height, has been utilized. One example of this is the circle at the Rockaway Beach end of the Cross Bay Bridge in Queens, New York. The rotary is elevated above the roadway which intersects the bridge approach and, as a result, is used only by traffic going on or off the bridge. Traffic on the intersecting road continues in a straight line without having to mingle with bridge traffic, and adequate turnouts are provided so that vehicles may turn off to the bridge or from the bridge to the road, without interrupting the main stream. Even when the span of the bridge is raised, there is sufficient space to store the bridge vehicles on the elevated circle so that traffic on the intersecting roadway can continue unhindered.

Another traffic circle that is unusual in that height was used for its efficacy is the three-level structure on the New Jersey Approach of the Lincoln Tunnel (Fig. 17). This development of the rotary principle goes beyond that of the aforementioned Cross Bay Bridge, for in this case the traffic on both the depressed approach and the intersecting avenue continues on tangent alinement without interruption. The circle, which is the intermediate level, is used only by vehicles turning from one roadway to another. The disadvantages of rotary traffic are overcome entirely on this structure because no weaving is required on the main roads, except possibly at the turnouts and, therefore, traffic is not slowed down. The radius of the circle is small so that comparatively little land is required, and yet it is large enough to accommodate the interchanging vehicles at the speed with which that operation is made. This type of grade separation is very well adapted to the intersection of express highways where the cost of land is high and where plenty of clearance between the two roads is available. It requires much less property than a clover-leaf structure and is much less confusing to drive upon, but a difference in elevation of approximately 40 ft between the main roadways is necessary.

The writer is grateful to Mr. Beretta for his valuable contribution regarding the approaches of international structures. The special problems which port-

of-entry inspection presents are complicated not only by the various types of traffic, each requiring a different type of inspection, but also by the multiplicity of agencies making these inspections. As Mr. Beretta suggests, how-

FIG. 16.—PEAK TRAFFIC AT NEW YORK APPROACH, HOLLAND TUNNEL, NEW YORK, N. Y.

FIG. 17.—THREE-LEVEL TRAFFIC INTERCHANGE AT NEW JERSEY APPROACH, LINCOLN TUNNEL, UNION CITY, N. J.

ever, these problems are capable of solution by the application of known principles of traffic design, together with those adopted from rail and water transportation.

Origin-destination studies should be especially helpful in designing international bridge approaches for they provide the key to the classes of traffic from which the degree of inspection required for each type may be determined. They also indicate the peak loads so that the capacity requirements may be estimated. With this information, the various traffic movements can be segregated among the inspection stations in a manner similar to the way in which they are distributed geographically from any other bridge approach. The remaining problem then is one of arranging and, if possible, consolidating the inspection facilities so as to require a minimum of conflict among the various traffic movements.

Mr. Hartley has made a valuable addition to the paper by his discussion on toll-lane capacities for small structures. Most of the toll stations at the various interchanges on the Pennsylvania Turnpike were arranged in the manner described by Mr. Hartley in Fig. 9(*c*).

In analyzing the capacity requirements for a small toll station, the low traffic-volume periods should be considered as well as the peaks in order that an excessive number of collectors will not be required. The layout of the Market Street Bridge in Harrisburg, Pa., is subject to criticism in this respect for it requires two men at all times to collect tolls, whereas one man could handle the traffic during the early morning hours if the booth arrangement permitted it. It is the writer's opinion that a double-faced booth should always be placed in the center of the toll plaza so that in low-volume periods the outer lanes can be closed and all traffic handled by one collector. Since the total traffic volume is likely to be less than 600 vehicles per hr even on a large structure for as much as 25% of the time, it is well to consider this period as much as the peak.

The toll-booth arrangement presented by Mr. Hartley in Fig. 13 allows considerable freedom of traffic movement but it is also subject to the same criticism of inflexibility in operation. A similar set of conditions prevailed on the layout of the Carlisle Interchange of the Pennsylvania Turnpike, but instead of being staggered, the booths were placed straight across the pavement, as shown in Fig. 18. Although this arrangement has only about 60% of the capacity of that shown in Fig. 13, it has the same advantages of minimum lateral movement, no extra graded width and, in addition, it may be handled by one man during low-volume periods. Furthermore, the separate booth for each lane not only permits faster collections in each lane but it also minimizes the inconvenience to the patrons since the tolls are always collected from the left-hand side of the vehicle.

The writer believes that Mr. Martin has perverted his splendid analogy with respect to wye connections that one "cannot mix two streams of automobiles as one mixes streams of water." A wye connection may be considered the same as a wye fitting in a pipe line; the smoother the connection is made, the less turbulence (traffic congestion) will be caused by the fitting. If vehicles must slow down or come to a halt at junctions, as Mr. Martin claims, why were the

accelerating and decelerating lanes provided on the approaches of the Highland Park Bridge (Fig. 14) which he cites as a good example of approach design? This same layout was used at the interchanges on the Pennsylvania Turnpike, and the accelerating and decelerating lanes were designed so that vehicles

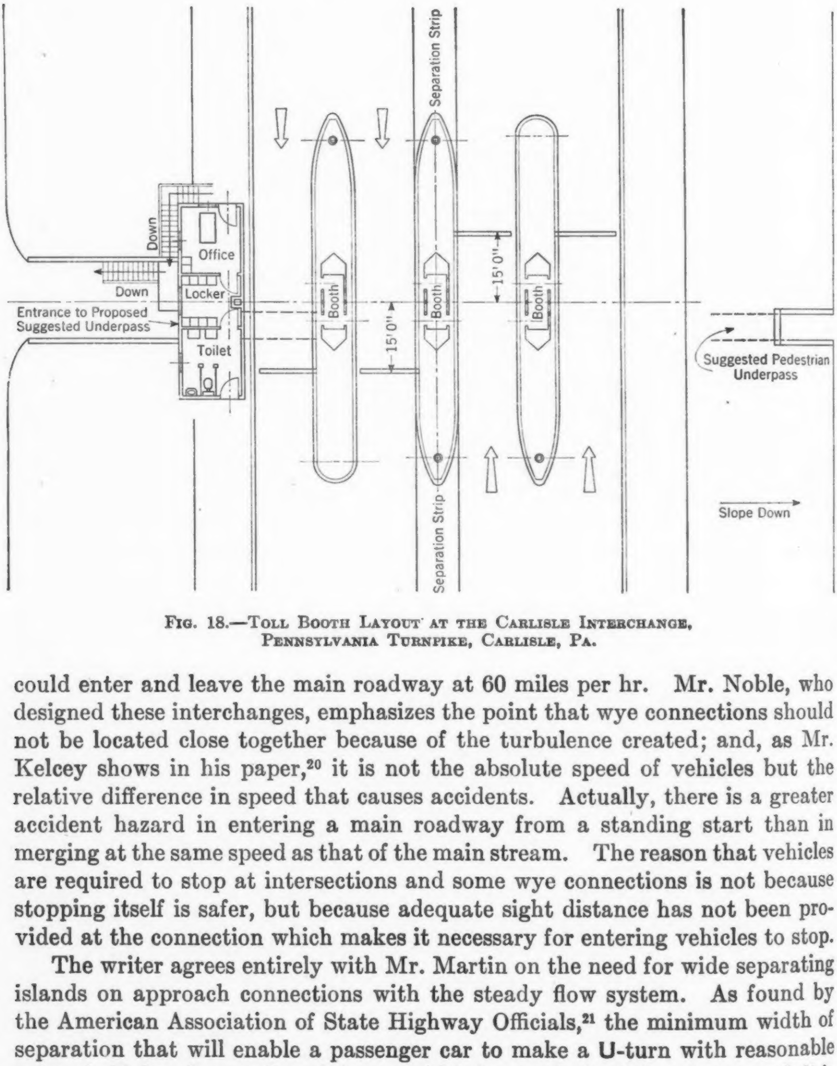


FIG. 18.—TOLL BOOTH LAYOUT AT THE CARLISLE INTERCHANGE, PENNSYLVANIA TURNPIKE, CARLISLE, PA.

could enter and leave the main roadway at 60 miles per hr. Mr. Noble, who designed these interchanges, emphasizes the point that wye connections should not be located close together because of the turbulence created; and, as Mr. Kelcey shows in his paper,[20] it is not the absolute speed of vehicles but the relative difference in speed that causes accidents. Actually, there is a greater accident hazard in entering a main roadway from a standing start than in merging at the same speed as that of the main stream. The reason that vehicles are required to stop at intersections and some wye connections is not because stopping itself is safer, but because adequate sight distance has not been provided at the connection which makes it necessary for entering vehicles to stop.

The writer agrees entirely with Mr. Martin on the need for wide separating islands on approach connections with the steady flow system. As found by the American Association of State Highway Officials,[21] the minimum width of separation that will enable a passenger car to make a U-turn with reasonable safety is 39 ft, whereas the minimum width for trucks is 71 ft. In general, it is desirable to make the separating islands as wide as possible. However, the cost of right of way, particularly in urban or suburban areas, frequently makes

[21] "A Policy on Highway Types," by Am. Assoc. of State Highway Officials, 1940.

it necessary to design these islands with less than the desirable minimum width.

Most sincere thanks are due to Mr. Noble not only for the valuable additions which he has brought to consideration but also for his efforts in applying some of the conclusions of the paper to practical test.

The writer agrees wholeheartedly with Mr. Noble on the importance of considering the influence of a bridge or tunnel approach upon adjacent property and the community. Only too often the broad, social and economic implications have been neglected, because of the desire to secure a low-cost solution to a bridge or tunnel approach problem. The effect of a bridge structure upon property value and the limitations of light and air imposed by such a structure were brought out fully in the controversy over the proposed Battery Bridge in New York City.

Mr. Noble's diagnosis of the factors involved in determining the ratio of length to width of plazas is very illuminating, although the writer does not agree that the use of $f = 0.15$ is high in determining this ratio. Mr. Noble states further that a friction of 0.25 to 0.30 frequently is developed at 15 or 20 miles per hr, but that at higher speeds less and less friction is utilized. Although it depends entirely on the design speed, the value given in the paper should be ample for most plazas, particularly since the table cited by Mr. Noble gives a permissible design speed of 65 miles per hr for $f = 0.15$.

Around this same point hinges the difference in the writer's and Mr. Noble's use of the tables for determining the forward distance required for lateral movement on plazas. The paper states that the design speed should be used in interpreting these tables, whereas Mr. Noble uses the posted operating speed, which is considerably lower, in applying the tabular values to the plazas of the George Washington Bridge and Lincoln Tunnel. The design speed on the plaza should be the same as that for the crossing proper for the reason given in the paper that "the speed of off-bound vehicles approaching the toll booths is the governing criterion." Mr. Noble is correct in his statement that on tunnel plazas the convergence should be completed 100 ft in front of the portal but it should be noted also that the length for convergence is measured from the ends of the toll islands rather than from their center line. Therefore, applying the tables to the New Jersey plaza of the Lincoln Tunnel, a lateral movement of 35 ft at a design speed of 50 miles per hr gives a minimum length of approximately 400 ft which compares favorably with the 450 ft that is provided on this plaza between the ends of the islands and the point of convergence. Similarly, on the George Washington Bridge toll plaza, the application of these tables at a design speed of 50 miles per hr gives a convergence length of 460 to 500 ft for a 48-ft lateral movement, whereas the actual length provided between the ends of the toll islands and the point of convergence is 490 ft. Considering the limitations of the tables and the instinctive design of these two plazas, both examples check the tables remarkably well.

Despite these comparisons, however, Mr. Noble is quite correct in his statement that the subject of convergence deserves additional study because of its economic importance. This writer cites the cost of the Lincoln Tunnel toll plaza at $8,500 per ft of width and it is a certainty that a comparably high value would be developed if the cost per linear foot of length were derived.

Although these tremendous costs tend to restrict the length and width of plaza to a minimum, there should be some assurance that sufficient space is being provided for vehicles to perform the required movements.

The limitations in using convergence tables such as those presented in the paper should also be recognized. These values cannot be applied to the approaches of the Pennsylvania Turnpike tunnels because these facilities are not plazas essentially. When Mr. Noble designed the Turnpike tunnel approaches he made an entirely different analysis which was more befitting the conditions under which vehicles are operating on them. The operating conditions on these approaches are similar to those at a main line switch on a railroad, whereas the plaza might be compared with a classification yard.

It is believed that Mr. Martin has misunderstood the writer's meaning of an approach in the statement that "Mr. Curtin has confused his discussion, to some extent, by treating both the approach and the approach connection the same." Without denying any confusion, it is the writer's feeling that they are the same, or at least that the connections are a component part of the approach. If a bridge or tunnel is to handle its full share of traffic with maximum convenience it should be designed as a whole, not as a patchwork of dissociated parts. Otherwise it never will fit together properly. The design speed and the various other design specifications should control the layout of the entire approach, including the connections. Although it may be necessary to use a different design speed on various parts of an approach to suit the operating or the physical requirements, it is most desirable to have the design unified and as uniform as possible throughout. Any changes should be made only with adequate transitions so that traffic may flow smoothly without suddenly being confronted with different operating conditions, for it is generally recognized that this is one of the primary causes of accidents.

Some disagreement prevails as to maximum lane capacity and the conditions under which it is developed. Mr. Noble takes exception to the value of 1,500 vehicles per lane per hr as being high in some cases, whereas Mr. Hartley cites an instance in which 1,800 vehicles per hr were counted in a single lane and also quotes values in excess of 1,800 per hr at speeds of 35 and 40 miles per hr from the Johanneson capacity formula. In addition, Mr. Martin states that the maximum capacity is reached at a speed of 23 to 25 miles per hr.

Unfortunately, there have been no studies which indicate conclusively the maximum capacity of a traffic lane, although enough work has been done to show definitely that many of the current formulas and notions concerning capacity are erroneous. From arbitrary formulas used in the past, the thought has prevailed that the maximum capacity of a lane is obtained when vehicles are moving uniformly at a speed of 23 miles per hr. Field studies conducted by the U. S. Public Roads Administration[22] have disproved this theory, however. They indicate that the theoretical maximum number of vehicles which one lane will accommodate in an hour is approximately 2,400 when the vehicles uniformly spaced are traveling at 31 miles per hr. Furthermore, these studies

[22] "Preliminary Results of Highways Capacity Studies," by O. K. Normann, *Public Roads*, Vol. 19, No. 12, February, 1939.

show that the theoretical maximum does not fall more than 20% below this value for speeds between 20 and 50 miles per hr.

It is definitely pointed out, however, in the conclusions of these field observations that such capacities are obtainable only in theory and that working capacities are well below these limits. It was more or less in recognition of these conclusions that the writer arbitrarily used a maximum of 1,500 vehicles per hr because, although there is no theoretical basis for it, this value is within the practical limits that have been observed. As a practical matter, neither maximum nor working capacities can be determined to such a nicety from consideration of vehicular speed alone, for the interference and differential speed between vehicles as well as the number of lanes and other design factors have a material influence upon capacity.

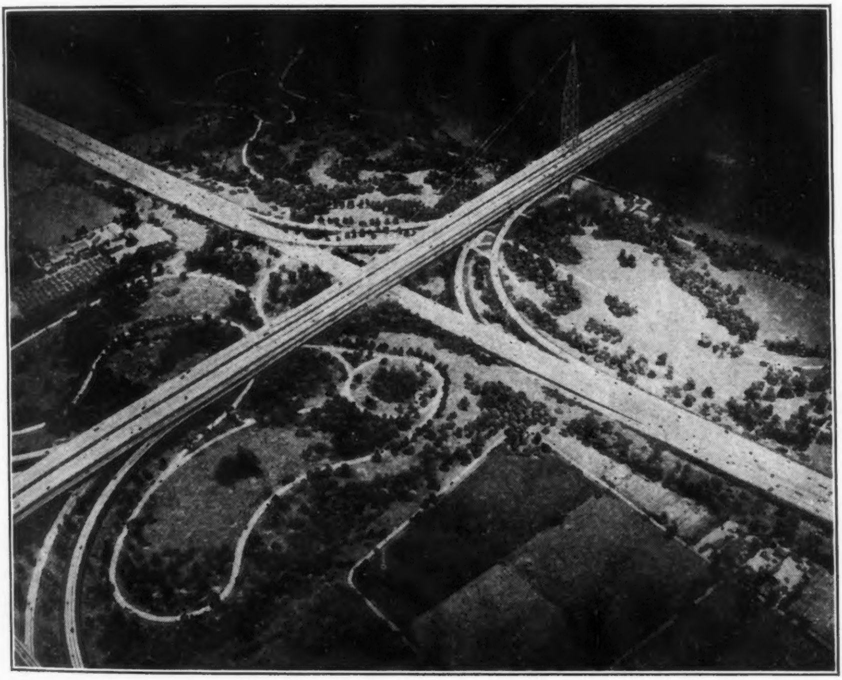


FIG. 19.—BRIDGE APPROACH PREDICTED BY ARTIST FOR TWENTY YEARS HENCE

Although it is not recommended necessarily, it is stimulating, nevertheless, to see the visionary conception of a bridge approach (Fig. 19) which Norman Bel Geddes predicts for 20 years hence.[23] The approach comes down gradually over an extensive area with long, flat curves branching from each of four levels. The principle is sound, but it is hoped that the profession will develop a more economical and less pretentious approach than that which this artist portrays.

[23] "Magic Motorways," by Norman Bel Geddes, Random House, Inc., New York, 1940.

Considering the progress made over the 20 years 1920 to 1940, it very likely will, for the fundamental principles have been established, examples are available to demonstrate them, and most of the factors involved have been evaluated. The experience of these twenty years is fruitful and all is ready now to be refined and molded to the desired objective. To this end, these meager efforts are dedicated—that some day the motorist in coming home at the weary end of a holiday will not be subjected to "that most dismal of all motoring experiences: driving back to the city across a bridge."[23]

Corrections for *Transactions:* In November, 1939, *Proceedings*, page 1541, four lines below Eq. 1, "a deceleration of 13.2 ft per sec^2" should read "a deceleration of 12.9 ft per sec.^{2}" In March, 1940, *Proceedings*, Fig. 9, on page 526, "(*a*)" should be labeled "(*b*)"; "(*b*)" should be labeled "(*a*)"; "(*d*)" should be labeled "(*e*)"; and "(*e*)" should be labeled "(*d*)."

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DISCUSSIONS

SEALING THE LAGOON LINING AT TREASURE ISLAND WITH SALT

Discussion

By Charles H. Lee, M. Am. Soc. C. E.

Charles H. Lee,[12] M. Am. Soc. C. E. (by letter).[12a]—Although he had hoped that the experience described in his paper would be more widely discussed, the writer is appreciative of the high quality of the three contributions submitted.

Professor Bodman draws from his extensive experimental work with agricultural soils in Western United States to develop the physicochemical background of the sealing action of salt. Of special interest to engineers is his statement of the relative adaptability to salt treatment of soils with similar mechanical analysis but with greatly differing clay constituents. He shows that highly siliceous clays of the montmorillonitic or bentonitic type can be rendered impervious much more readily than those of the kaolinitic type, which contains much greater percentages of iron and aluminum. The clay used in the lining of the Lagoon at Treasure Island was of the siliceous type.

In passing, it is to be noted that this chemical classification of clays refers to the clay particles that act as negatively charged cores or anions, whereas the writer's classification as "calcium" or "sodium" clay was with reference to the surrounding and much smaller positively charged cations. Both classifications are required for a complete chemical discription of a clay.

With reference to the permanence of a reservoir lining rendered impermeable with salt, Professor Bodman states that occasional replenishment of sodium ions might be necessary to maintain impermeability, not alone with respect to highly calcareous or saline waters, but even for water of low mineral content. Water used to fill and replenish the Lagoon throughout the life of the Exposition was obtained from the San Francisco Water Department. Typical analyses of this water at various times of the year indicate a water of low mineral content, with calcium predominant (see Table 14).

Note.—This paper by Charles H. Lee, M. Am. Soc. C. E., was published in February, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1940, by Messrs. G. B. Bodman, and John D. Watson; and October, 1940, by John W. Pritchett, Assoc. M. Am. Soc. C. E.

[12] Cons. Hydr. Engr., San Francisco, Calif.

[12a] Received by the Secretary November 6, 1940.

During twenty-one months of service, the Lagoon showed no tendency to lose its initial seal. It is unfortunate that the Lagoon must be destroyed because longer service would have furnished valuable information on this point.

TABLE 14.—MINERAL ANALYSES, SAN FRANCISCO WATER DEPARTMENT (Parts per Million)[a]

Date (1937)	Solids[a]	Si	Na	Ca	Mg	Fe	Al	H CO_3	CO_3	SO_4	Cl
February 10[b]	34.0	2.9	9.4	10.9	4.2	0.3	0.25	56.0	0	7.2	8.0
May 11[c]	91.0	6.7	7.0	13.1	5.3	0.19	0.49	61.0	0	11.7	9.0
November 15[c]	124.0	2.9	11.0	19.4	7.6	0.1	0.26	90.0	0	15.6	11.0

[a] Solids = dissolved solids; Si = silica and insoluble matter; Na = sodium (including potassium, K); and Fe = iron (dissolved); other columns are as shown. [b] Sample from the surface of Crystal Springs Reservoir. [c] Sample from the water line at Millbrae, Calif.

Mr. Pritchett's contribution to the discussion is of especial interest as it illustrates, strikingly, two points of great importance in the use of salt as a sealing agency. To begin with, the predominating soils in the region where Mr. Pritchett is working are of the red and yellow podsol type composed of lateritic material with a "silica sesquioxide ratio" of approximately 2. Clays associated with this type of soil are of the kaolinitic type. Extensive areas of this soil occur in eastern Texas, western Louisiana, southern Mississippi and Alabama, and central Georgia where it has been termed the Norfork-Ruston soil series by the Bureau of Soils, U. S. Department of Agriculture.[13] Color description of clay test samples listed in Table 12 show a predominance of reds and yellows, and it is probable that tests were all made on clays of this type. As stated by the writer, and also by Professor Bodman, sodium treatment has little effect upon the physical properties of such clays. This is probably the explanation for the failure of the salt admixture to produce a degree of disintegration in clay balls when submerged in water, differing from that of balls mixed with fresh water.

The other point involves the proposed storage of salt water in reservoirs sealed with salt-treated clay. The writer's experience at Treasure Island was that less than 50% of the final sealing effect was obtained by use of salt water alone, the fresh water being necessary to leach out excess sodium before the full effect of salt treatment was obtained. Laboratory tests also show the necessity of fresh-water leaching following salt application before permeability is effectively reduced. Professor Bodman states that "Long-time storage of high-lime water, or of highly saline water, would not prove successful." Even if a satisfactory clay were found in local deposits of alluvial materials along the Neches River, it is doubtful whether salt treatment would solve the problem of storage of such highly saline waters as are encountered in the east Texas oil field. Compaction or some method of soil stabilization would seem to be the only answer in this region.

In opening his discussion, Professor Watson states that "* * * if the clay had been properly applied, the salt would not have been needed." The writer is in entire accord with this statement, assuming that "properly applied" refers

[13] "Soils and Men," *Yearbook of Agriculture,* U. S. Dept. of Agri., 1938, p. 1069 and inset map

to moisture control and method of rolling. Acceptance of the clay by the writer was predicated upon placing with uniform moisture at optimum moisture content and rolling with a sheepsfoot roller. Neither requirement was followed in actual construction, however, for reasons beyond the writer's control. It is the writer's opinion that a sheepsfoot roller could have been used satisfactorily to compact the clay layer, without breaking through into the sand, either by increasing the thickness of the first clay layer or by stabilizing the sand subgrade by sprinkling it with water before spreading the clay. As for moisture control, its use is now almost universal in this type of construction, and methods for its accomplishment are a matter of routine. The salt treatment was prescribed only after the completion of the work, when a test of the poorly compacted clay membrane disclosed its leaky condition.

The writer does not agree fully with Professor Watson's statement that it was an error to buy a soil on the basis of particle size. Although the specifications, as prepared by the Construction Division of the State Department of Works, might well have provided for a permeability test, such test would have been subject to the same limitation as was the mechanical analysis—namely, the failure of the clay material, as actually placed, to conform in density with the laboratory test samples at ideal density. The writer has directed permeability tests for many years, using Darcy's law, and has found wide differences in the same material with differing degrees of density.

With regard to Mr. Hazen's precautionary statement, referred to by Professor Watson, careful reading of the original discussion shows that he had in mind the indiscriminate use of mechanical analysis in conjunction with the Hazen formulas for flow of water through filter sands and other clean granular materials of uniform grain size. He states that these formulas and their constants were developed for a special type of filter sand and do not apply to the general run of natural earth materials used in engineering construction.

In another paper Mr. Hazen states,[14]

> "It is to be noted that the formula was never intended to apply to clays, hardpans, soils, and other materials. The effort to apply it to such materials is not to be encouraged, and the results are not to be depended on."

It was not the mechanical analyses, as such, that he warned against, but the indiscriminate use of his formulas which, as a preliminary step, involved mechanical analysis.

In discussing these matters with Mr. Hazen many years ago, in connection with the computation of underflow through the narrows of an alluvial-filled river valley, the writer also recalls that he advised against the application of his formula to natural alluvial materials but did not advise against mechanical analysis as it might throw light upon permeability as affected by distribution of particle sizes. In fact, a very important element in the Hazen formula is the constant "*c*" which has wide variation, depending largely upon the variations in proportions of large and small particles as expressed by the uniformity coefficient obtained from the results of mechanical analysis.

[14] *Transactions*, Am. Soc. C. E., Vol. LXXIII, September, 1911, p. 201.

It has been the writer's experience that those natural earth materials whose distribution of particle sizes conforms closest to the general grading equation

$$P = \left(\frac{d}{D}\right)^n \dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

(in which n lies between 0.25 and 0.40) are the most impermeable, and that material substantially following this equation with a degree of fineness sufficient to include at least 3% of clay, or poorly graded material containing at least 25% of clay, when thoroughly compacted, is practically, if not fully, watertight.[15] Mechanical analyses of samples submitted by the contractor indicated that the selected clay fulfilled requirements of the grading formula, Eq. 1, and personal handling of the clay by the writer gave added confidence as to its impermeability if properly compacted.

The writer's statement of conditions during the two months' period between the completion of the placing of the clay membrane and the first flooding is probably incomplete, as Professor Watson states that the clay "was subjected to two months of severe drying, and * * * the entire layer was fissured with shrinkage cracks." As a matter of fact, the entire surface of the clay was sprinkled once each day throughout the two-month period. The rate of evaporation, augmented by afternoon trade winds, was so great, however, that the surface dried out before the day was over and thus gave opportunity for cracking. Serious cracking was limited to the top layer, cracking in the bottom layer being superficial. The writer does not agree with the statement that the "* * * slaking accounts fully for the high rate of seepage and for the complete softening of the clay layer." It certainly was a contributing factor, but the basic cause was inadequate compaction. The daily sprinkling would probably have been adequate if the material had been compacted to the density possible at optimum moisture content. It had been planned to flood the clay layer immediately after completion, but unforeseen delay in the execution of another lagoon contract made this impossible.

With regard to the relative rate of downward progress in softening of the clay with fresh and with salt water, tests were made during the first six days of submergence by salt water, and softening did not occur to the same depth as with fresh water. In fact, the depth of softening at the sixth day was 2 in., or less, and was not materially greater at sixty-five days. The rate of compaction under a seepage pressure amounting to 1 lb per sq in., as computed by Professor Watson, is so slow that many years would have been required to harden the clay, rather than a few weeks, as he suggests. Under certain conditions, sealing of leaky reservoir bottoms by seepage pressure, by accumulation of a skin layer of fines, or by other means, has been experienced by the writer. Such sealing, if it occurs, requires a period of years for its consummation, whereas expositions come and go within a period of months. Application of effective salt treatment requires only a few weeks. In practical engineering the

[15] "Selection of Materials for Rolled-Fill Earth Dams," by Charles H. Lee, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1.

means for accomplishment must be adapted to the end in view. The engineer must select those forces of nature which will meet schedule as well as be effective.

Returning to Professor Watson's opening statement, he questions the utility of the salt method in view of the possibility of securing impermeability by compaction. Although it is true that impermeability of a well-graded material can be obtained by compaction, this is not so true of clay materials lacking the sand fractions. For such materials, for the reduction of hazard with compacted clay materials having the sand fractions, and for remedial work such as that described by the writer, the salt method unquestionably has a future.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

NORRIS DAM CONSTRUCTION CABLEWAYS

Discussion

By R. T. Colburn, M. Am. Soc. C. E., and L. A. Schmidt, Jr., Assoc. M. Am. Soc. C. E.

R. T. Colburn,[13] M. Am. Soc. C. E., and L. A. Schmidt, Jr.,[14] Assoc. M. Am. Soc. C. E. (by letter).[14a]—This will refer to the discussion by Mr. Foster, in which he gives the power requirements for moving the cableway towers of Conchas Dam. Since one of the cableways at Norris Dam was erected, and has poured all of the concrete at Hiwassee Dam subsequent to its dismantling at Norris Dam, the writers have been able to check their measurements on the pulls against the traversing anchorages at the latter project. Fig. 17 shows the recorded pulls on the anchorage for periods when the towers stood at rest, when they were starting, and when they were moving.

No clearly defined points were determined to differentiate between the head tower and the tail tower; nor was there any consistency in the results obtained for immediate movement or after long periods of rest, and therefore the total number of readings taken for each position has been plotted and the ranges of pulls are indicated between the outside limits of the readings taken. Fig. 17 indicates that, similarly to Norris Dam, pulls of approximately 70,000 lb may be expected.

As to the horsepower requirements, several readings of the voltage and current during starting and running indicate that approximately 100 hp was required to start the head and tail towers, and that approximately 60 hp was consumed when the tower was moving.

The life of the 3-in. locked-coil track cable used at Norris and Hiwassee dams far exceeded the expectations of TVA engineers. A total of 427,090 cu yd of concrete was traversed over this cable at Norris Dam, and approximately 795,000 cu yd were hauled at Hiwassee Dam. Besides this, 33,272 tons of other

Note.—This paper by R. T. Colburn, M. Am. Soc. C. E., and L. A. Schmidt, Jr., Assoc. M. Am. Soc. C. E., was published in December, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1940, by J. S. Foster, Esq.; April, 1940, by Gordon H. Bannerman, M. Am. Soc. C. E.; June, 1940, by Messrs. Walter F. Weber and Blair Birdsall, and G. E. Cate; and November, 1940, by Adolph J. Ackerman, M. Am. Soc. C. E.

[13] Constr. Plant Engr., TVA, Knoxville, Tenn.

[14] Designing Engr., Constr. Plant, TVA, Knoxville, Tenn.

[14a] Received by the Secretary October 18, 1940.

materials were moved by this cableway at Norris Dam and probably twice as many tons at Hiwassee Dam.

Maintenance, consisting of greasing and periodically turning the cable, was a large factor contributing to the longevity of this excellently built cable. The careful design of the carriage for even distribution of wheel loads to reduce bending stresses in the cable is also believed to have contributed to the lasting qualities of the cable.

The discussion by Mr. Ackerman has added greatly to the scope of the original paper by giving data on a number of older, and one of the more recent installations—namely, Shasta Dam. It is gratifying to know that advantage

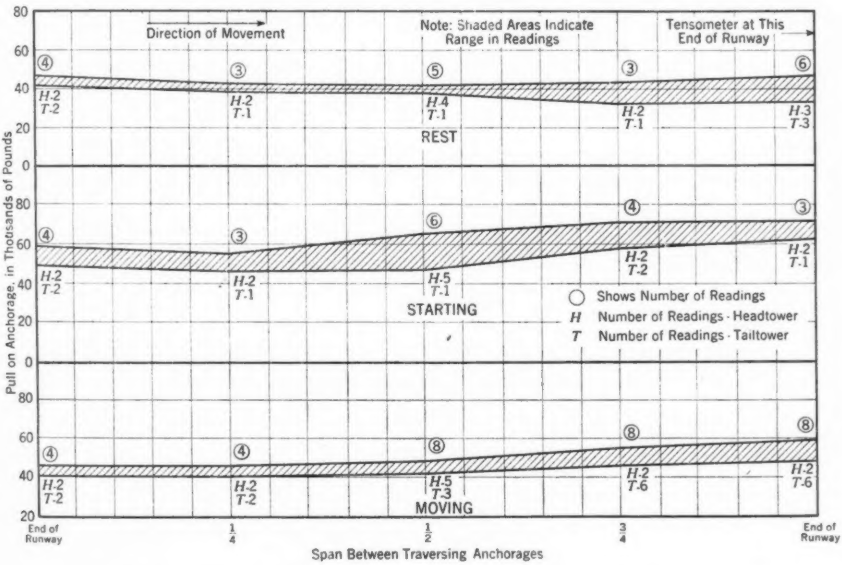

FIG. 17.—RECORDED PULLS ON TRAVERSING ANCHORAGE, HIWASSEE CABLEWAY

was taken of the results of the tests at Norris Dam in the design of this cableway installation. The discussion by Mr. Cate provided a very complete analysis and derivation of the formulas used by the writers. The additional comments in the discussion given by Messrs. Bannerman, Weber, and Birdsall were a valuable addition.

Acknowledgments.—In closing this discussion the writers wish to make the following acknowledgments: Tests were conducted by Douglas McHenry, TVA, at Norris Dam and Hiwassee Dam, who also made all the calibration tests on the tension indicator discussed under "Tests," and provided the discussion of the use of this instrument in this text. T. S. Whitehouse, construction plant electrical engineer, provided the discussion of the electrical features; and P. H. Kline, construction plant operations engineer, furnished operating data.

Special acknowledgment is made to Mr. Ackerman, formerly construction plant engineer, TVA, who supervised the design, installation, and operation of the cableway; to S. A. Parish, of the American Steel and Wire Company, and to Mr. McHenry, for suggestions offered by them after reviewing the paper. Barton M. Jones and Ross White, Members, Am. Soc. C. E., were construction engineer and construction superintendent, respectively, on Norris Dam.

The TVA design check of the cableways, the cableway layout, and runway design were made by Mr. Colburn; and the analysis and comparison of the test data by Mr. Schmidt.

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DISCUSSIONS

CHICAGO RIVER CONTROL WORKS

Discussion

BY H. P. RAMEY, M. AM. SOC. C. E.

H. P. RAMEY,[17] M. AM. SOC. C. E. (by letter).[17a]—The writer wishes to express his appreciation to those who have discussed this paper and have thereby called attention to certain items which were perhaps inadequately described.

Mr. Niederhoff has called attention to the fact that the Government breakwater and North Pier, forming enclosure walls of the Chicago River Control Works project, were not impervious, watertight structures. The rock-filled timber cribs forming these structures offered so little resistance to the flow of water from the lake to the river that it was impossible, except under conditions of heavy flow, to maintain water levels in the river appreciably lower than in the lake. During the summer of 1939 a careful survey was made of the Government breakwater and North Pier to determine the extent of work necessary to render these structures watertight, and on August 15, 1940, a contract was awarded for such work. The contract provides for driving a row of steel sheet piling 10 ft from, and parallel to, the landward face of the cribwork; tying the sheet piling to the concrete cap over the cribs, with steel tie rods at proper intervals; filling the 10-ft space between cribs and sheet piling with clay (from Chicago subway excavation); and protecting the row of steel sheet piling with stone fill (on the landward side) to the top of the piling. This work was completed in November, 1940, and added $127,000 to the $2,704,000 cost mentioned in the paper.

Messrs. Niederhoff and DeYoung have both mentioned the spalling of concrete, from impact by boats, and both have made excellent suggestions. Armor, wales, and fenders were considered, but all were ruled out at the time because of cost. It is the conviction of the writer that armor and either wales or fenders will be installed ultimately. The plans were reviewed by a PWA Review Board, interested in keeping costs within an allocated sum. The

NOTE.—This paper by H. P. Ramey, M. Am. Soc. C. E., was published in January, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1940, by Messrs. A. E. Niederhoff, R. P. V. Marquardsen, Isaac DeYoung, John W. Woermann, Edward Soucek, W. C. Weeks, and Henry R. King.

[17] Asst. Chf. Engr., San. Dist. of Chicago, Chicago, Ill.

[17a] Received by the Secretary October 25, 1940.

concrete is of good quality. In many places where concrete has spalled, the particles of coarse aggregate (granite gravel) have split as readily as the remainder of the concrete. There has been no crumbling. The placing of this concrete was inspected by the Sanitary District, the PWA, and representatives of the Army engineers.

Mr. Marquardsen, who was one of the principal structural designers on this project, has contributed a valuable discussion of many of the details of the work. His discussion of the operating machinery has probably answered a question suggested by Mr. Niederhoff.

Mr. DeYoung has suggested that a monolithic concrete floor would have been better than the loose slab floor, particularly if the lock should be unwatered. The writer agrees with this view. Mr. Leffler, who designed the lock, favored such a concrete floor, placed "in the dry," to assure accurate construction and to prove the stability of the lock walls.

Mr. DeYoung prefers gravel fill for the cells because of its compactness and quick settlement. His suggestion of timber piles within the cells, to provide shear, is very good. Mr. Woermann obviously prefers stone fill, an honest difference of opinion. Mr. Woermann has reviewed some of the considerations given to various types of lock gates, which ultimately led to the adoption of the triangular-shaped sector gates. The distribution of water currents shown in the tests of the model of this Swedish installation checked conclusions reached by the engineers of the Sanitary District.

Mr. Soucek's discussion of the hydraulic formulas is interesting and constructive. His simplified formulas will give results sufficiently accurate for any practical purpose. He points out that the adoption of the Francis formula for the "weir" term may not be correct since the element of crest contraction is entirely lacking. Apparently he is of the opinion that the product of the weir coefficient 0.62 and the over-all coefficient 0.95 conforms closely to that obtained in tests. The coefficient 0.95 as used in the formula, however, was introduced to take care of side contractions. Probably more in line with what Mr. Soucek had in mind is the derivation of the formula as presented in Mr. King's discussion in Eq. 17. Incidentally, Mr. King is entitled to much of the credit for deriving and checking the formulas for discharge to and from the lock. No allowance was made for flow under the gates when they are in a partly opened position. The head differential is so small by the time the gates have opened sufficiently wide to clear the bottom sill substantially that the flow beneath the gate is practically negligible as compared to the flow through the three main orifices.

Mr. King mentioned the accumulation of ice and floating debris carried into the gate recess, between the straight face of the gate leaf and the curved face of the recess. One plan suggested to avoid this difficulty is to attach falsework to the front of the gate leaf in the shape of a circular arc fixed horizontally, with its center at the gate hinge, and extending vertically down to about 4 ft below the water line, or below the line of floating ice. This would obstruct the upper parts of the side orifices and slightly lengthen the time of filling and emptying the lock. No conclusion has been reached on this matter

as yet. Ice troubles have been avoided by frequent operation of the gates for the specific purpose of breaking up the ice.

Colonel Weeks, who was United States District Engineer in Chicago during the years when the Chicago River Control Works project was the subject of discussion and reports in the controversy regarding lake levels between the State of Illinois and other Great Lakes states, has contributed a summary of certain events which preceded the adoption of the present plan. His views of the advantages and disadvantages of this structure in Chicago Harbor are sound.

It is extremely gratifying to the writer to have been able to present a paper which has elicited such constructive criticism and so much friendly comment. The labor has not been in vain.

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DISCUSSIONS

THE PRACTICE OF STATE HIGHWAY DEPARTMENTS IN THE DESIGN OF ABUTMENTS

PROGRESS REPORT OF A SPECIAL SUBCOMMITTEE OF THE COMMITTEE OF THE STRUCTURAL DIVISION ON MASONRY AND REINFORCED CONCRETE

Discussion

By Messrs. Leslie R. Schureman, and H. G. Overholt

Leslie R. Schureman,[5] Assoc. M. Am. Soc. C. E. (by letter).[5a]—The writer has studied the Committee's Report and the appended questionnaire with great interest. That there is, and has long been, a genuine need for the clarification of many of the issues involved in the design and construction of highway bridge abutments is evidenced in the considerable differences in practice uncovered by the questionnaires. It is to be hoped that a thorough analysis and discussion of the data presented in this Report will help materially in removing much of the guesswork now existent in this phase of bridge design.

With regard to the loads that may act upon the abutment, the writer would suggest that it is not always safe to neglect entirely the horizontal forces arising from thermal expansion and contraction of the superstructure. Although this force is not mentioned in the Report, it may achieve major importance in the case of high abutments combined with high friction superstructure bearings. Obviously the force will be of low magnitude if the bearings have been well designed, but unfortunately this has not always been done. The writer feels that at least some of the abutment troubles encountered in past years can be attributed, either partly or wholly, to superstructure bearings which had high friction initially or which developed high friction with age.

With regard to types, it is interesting to note that the trend is away from the gravity section for concrete abutments. This trend would seem to support the writer's opinion that this type, a carry-over from the stone masonry wall, can seldom be justified economically in view of the inherent economy of steel

Note.—This Report was published in June, 1940, *Proceedings*. Discussion on this Report has appeared in *Proceedings*, as follows: September, 1940, by J. Wayne Courter, Assoc. M. Am. Soc. C. E.; and October, 1940, by Messrs. E. W. Wendell, and Stanley Levitt.

[5] Asst. Prof. of Civ. Eng., School of Eng., Princeton Univ., Princeton, N. J.

[5a] Received by the Secretary October 23, 1940.

and concrete in combination. To the discussion regarding the spill-through type of abutment, the writer would add only that, strictly from the viewpoint of hydraulic efficiency, it is far superior to the closed type.

The Report indicates that considerable attention is being given to foundation exploration and increasing attention to soil analysis. The importance of a dependable picture of subsurface conditions unquestionably demands a thorough investigation and analysis of the foundation upon which the abutment is to rest. In this connection, the writer has found wash borings to be so misleading in some cases as to be worse than no borings at all, and certainly of much less value than the sounding rod. At the present time, there is a real need for a reliable means, within the reach of the average bridge engineer, of relating the results of subsurface exploration to reasonably true load-carrying capacity. In view of the fact that such a means is essential to the efficient design of abutments, a direct effort in this direction by those engaged in soils study is clearly indicated.

In initiating and carrying forward a detailed study and discussion of this subject, the Committee is rendering a valuable service to a substantial part of the membership and is certainly to be highly commended for it.

H. G. Overholt,[6] M. Am. Soc. C. E. (by letter).[6a]—It is gratifying to note that the Report of Professor MacLean's Committee is submitted as a Progress Report. Further work and reports in continuation of the subject will be welcomed by the profession.

Soil Mechanics.—Soil mechanics is certainly the great blind spot in the special field of substructure design. Nature provides the true or ultimate foundation for all of man's structures, and the filling materials are raw, natural products. Thorough study of these component parts of the complete edifice is just as necessary as is control of the processed material entering into the structure. To date, this new science has produced little of direct quantitative nature. A personal impression is that soils engineers should lead the way to formulation of practical means of application of their analytical and field data.

Papers and studies coming to hand so far have revealed startling facts on pressure distribution and kindred features but have made no approach to design methods for the usual types of structures. The impression given is that settlements may be predicted, but only after an elaborate study of formations to a considerable depth. Means of estimating normal (lateral) pressures and parallel (shearing) forces on the planes set up for design purposes would be of much greater value, and it would seem that workers in the field of soil mechanics could produce more along this line than has been done.

The preliminary draft of the newly revised American Association of State Highway Officials (AASHO) Code sets extremely broad limits for bearing power of soils and establishes a minimum design lateral pressure of 30 lb of equivalent liquid. New and valuable rulings show progress over the current (1935) edition; nevertheless, the code, as revised, remains conservative, and advisedly so. That it has been thought well in our national codes to maintain a goodly margin

[6] Associate Bridge Engr., State Highway Dept., St. Paul, Minn.

[6a] Received by the Secretary October 28, 1940.

for the factor of ignorance, rather than for the factor of safety, is discernible throughout the substructure sections. This situation furnishes an opportunity for soil mechanics work.

Buried Abutments.—The point was not brought out in the Report that the "openings" in such abutments are the net spaces between the footings of the columns or legs, rather than the spaces between the legs themselves. For design purposes, all filling material over a footing is treated as if it were, say, 100-lb concrete, placed with the structure. This fact points directly to Mr. Wendell's practice of using this type only on strata where the footings can be made relatively small in area. Plans have been seen in which the spaces between footings were one half or less of the height of the openings, and in two cases the footing was continuous over the entire length.

It is believed that the extent of "drag" is not commonly realized. Grain-elevator design is illuminating on this point: A wood crib elevator may be compressed in its height by 2 ft and more on the first drawdown of the grain. In Minnesota there has been more damage from the drag of subsiding fills than from lateral pressures.

Piling through High Fills.—Mr. Martin of Kansas comments on an anomaly that has caused some confusion and indecision. In the current, highly compacted fills, wood piling cannot be driven to any depth. Piles, driven with much difficulty, end within the new fill or penetrate below it only a few feet. What, then, has been gained by the attempt to drive long piles? Would it not be wiser to drive piles of 20-ft or 25-ft lengths, rather than pound away on longer piles, since subsidence of the fill will carry the abutment down with it in either case?

The alternative, mentioned by Mr. Martin, of using long steel piles has been used in those cases where hard formations can be reached with a reasonable length of pile. In other cases, where substantial refusal cannot be obtained, it probably would be better to use wood piles of nominal length and to give more attention to trenching and rolling the subsoil.

Questions for Further Study.—Of the six questions proposed, the writer feels that the sixth one, dealing with buoyancy, is satisfactorily resolved by the proposed code revision to allow for the full buoyancy effect in all cases.

Question 3, dealing with batter piles, should be broadened to include the development of methods of complete analysis for the numerous cases in which circumstances do not warrant the common assumption that all lateral force is unloaded to the ground or fill at the elevation of top of piles. The method proposed by C. P. Vetter,[7] M. Am. Soc. C. E., has been found to be quite simple in application, if the piles are treated as hinged structural members, with their bending resistance ignored and with no account taken of lateral earth pressure against them.

Question 4, dealing with pile-driving formulas, might well be extended in scope to include pile-driving procedure and interpretation of penetration records. Means of detecting and avoiding damage to both wood and steel piles under hard driving is also worthy of investigation.

[7] "Design of Pile Foundations," by C. P. Vetter, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 758.

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DISCUSSIONS

MASONRY DAMS
A SYMPOSIUM

Discussion

By Homer M. Hadley, Assoc. M. Am. Soc. C. E.

Homer M. Hadley,[111] Assoc. M. Am. Soc. C. E. (by letter).[111a]—The papers of this Symposium constitute an admirable presentation of both theory of design and construction practice which have obtained during the greatest dam building period in history. Unprecedented in height, great size, and number of large structures, the dams of this time have received the most careful, thorough, and painstaking engineering thought and attention—to which fact they will give witness for centuries and millenniums to come.

For satisfactory performance the concrete (Mr. Tyler states) must possess strength, weight, durability, impermeability and "continuity [that is, freedom from cracks] in order that the structure may act according to assumptions of design." The other authors either directly or implicitly indorse this judgment which unquestionably is the widely held and practically unanimous opinion among engineers today. Nevertheless it would appear that continuity might well be accepted as satisfactory even if it were of relative rather than of absolute degree and that impermeability of structure might be obtained, advantageously and economically, in numerous cases by means other than those of present practice.

The enemy of continuity in concrete is the crack which generally has its origin in thermal and moisture-content changes. Because of internal shrinkage, that which was whole and continuous becomes divided into separate parts by fissions or cracks. The structural consequences of such cracking may be serious or not, depending upon the width of the crack, the shape, form and direction of the surfaces of cleavage, and upon the physical structure, aggregate particle size, and characteristics of the concrete itself. A crack, $\frac{1}{16}$ in. wide,

Note.—This Symposium was published in May, 1940, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: September, 1940, by Messrs. William P. Creager, J. R. Shank, George R. Rich, Robert A. Sutherland, Ross M. Riegel, Paul Baumann, W. A. Perkins, L. J. Mensch, and Lewis H. Tuthill; October, 1940, by Messrs. F. A. Nickell, Leslie W. Stocker, Barton M. Jones, P. E. Gisiger, Joseph A. Kitts, S. O. Harper, and R. F. Blanks; and November, 1940, by Messrs. Berlen C. Moneymaker, A. Warren Simonds, and W. J. E. Binnie.

[111] Regional Structural Engr., Portland Cement Assoc., Seattle, Wash.

[111a] Received by the Secretary November 2, 1940.

developing on what approximates a geometrical plane surface in a mortar having $\frac{1}{4}$-in. maximum-size aggregate would produce vastly different effects on stress distribution than would the same $\frac{1}{16}$-in. crack developing with warped irregular surfaces in a concrete made with 6-in. or 8-in. maximum-size aggregate. In the first case there would be practically complete loss of shearing strength in the plane of the crack, a complete loss of tensile strength perpendicular to the crack and, until the crack closed again, a complete loss of compressive strength. In the second case there would be only a slight loss of shearing strength, and even perpendicular to the crack at any point, due to aggregate interlock, a certain amount of tensile and compressive strength would persist. Let it be emphasized here that cracks in mass concrete do not form plane surfaces; they bend and twist to form warped and highly irregular surfaces which themselves interlock independently and additionally to the aggregate interlock. If, then, with normal dam concrete, cracks are not so closely spaced as to destroy columnar action in the sections between them, it is difficult to see anything particularly grievous structurally to result from them. It is to be recognized, of course, that there are cracks and cracks, some of which may truly be of a serious nature; but there are also others which are so utterly trivial and insignificant as to warrant little or no "dither" about them or the incurring of expense to prevent their formation.

There is the possibility, of course, that cracks in dams may become filled with water under pressure. In the case of cracks perpendicular to the axis it is generally agreed that no harm would result if such a thing did happen, but water under high pressure in cracks parallel to the axis would produce lateral thrusts tending to disrupt the mass by splitting off successive laminæ from the downstream face. Although such action may be assumed, of course, a much more cheerful and probable assumption to make is that nothing of the kind will occur. Cracks parallel to the axis of the dam are intersected at regular intervals by cracks perpendicular to the axis, which latter vent to the atmosphere at the downstream face of the dam. Consequently when and if pressures do tend to build up they relieve themselves via the perpendicular cracks. A wide field of assumptions can be made regarding the extent, direction, and pattern of cracks that may develop. However, certain things are inherently more probable than others. Because the length of a dam is almost certainly greater than its thickness, cracks transverse to its axis are inherently more probable than cracks parallel to the axis despite the fact that escape of heat and moisture from the mass is outward from the surfaces and therefore transverse to the axis. Therefore, to assume cracks parallel to the axis without the simultaneous existence of cracks perpendicular to the axis is to assume a possibility, yet scarcely a probability.

That water pressures in cracks, if they do tend to develop, find relief without causing damage, and that this is not mere wishful thinking, is attested by definite evidence. Prior to the present dispensation of crackless concrete, numerous dams were built which incorporated the errors now painstakingly avoided. It is true that these older dams are not of the magnitude of those of the present day, but they are fair-sized scale models of present dams. Being built with high-heat cement, in large masses, with uncooled mixing water and

aggregates, without artificial cooling, and without the careful foundation treatments now provided, they developed cracks perpendicular to their axes and likewise parallel to their axes. Opportunities for water under high pressure to penetrate these cracks have been quite as favorable as any to be found today. Despite these opportunities for trouble and disaster (if such indeed they are), not one of them that the writer has ever seen or read about has suffered in consequence. These old dams present many a warning against watery, sloppy concrete, against segregation, against careless construction joints, against concrete so porous or weak that it is unable to withstand weathering or ordinary abrasion. Nevertheless, as far as offering any evidence corroborating fears about harmful high water pressures developing in ordinary shrinkage cracks, that is the thing they signally fail to do.

In many places freezing occurs, and then the question of possible damage from the freezing of water in cracks arises. The writer has seen a considerable amount of frost damage in porous concrete. This has been characterized by general surface scaling with increased disruption at horizontal construction joints with their laitance. On the other hand, where the concrete has itself been of a quality to be classed as frost resistant, he does not recall having seen damage at cracks. A very excellent example of such contrasting behavior was seen several years ago in a box flume on the Owens Valley aqueduct of the City of Los Angeles, Calif. This elevated box flume was a reinforced structure built with standard portland cement. The approach conduit sections were built with the tufa cement (puzzolanic) generally used throughout the aqueduct.[112] In about twenty years of occasional freezing and thawing, leaking cracks in the sides of the approach sections had spalled considerably around the edges of the cracks and over adjoining surfaces, whereas leaks in the box-flume structure along the construction joint between the bottom slab and the side walls had spalled not at all. This is not an analogous case, of course, to the freezing of water in cracks parallel to the axis of a dam; but instances of damage from such a cause are, to say the least, not of common record.

The writer never saw St. Francis Dam either before or after the catastrophe, the cracks in which are frequently referred to; but he has a vivid recollection of being shown certain hard fragments of foundation material taken from the very place in the stream bed upon which only a few days before the St. Francis Dam had stood. The behavior of the fragments of this "bedrock" was this: Dropped into a glass of water, they emitted some tiny bubbles and quickly softened, lost their shape, and flattened into a thin layer of mud in the bottom of the glass! If such was the behavior of these fragments pried from the St. Francis "bedrock" it seems quite possible that water stored behind the dam might, slowly, have worked its way through softened material beneath the rudimentary cutoff wall and beneath the dam and at last have piped through—first a trickle, then a jet, then a bursting stream, and finally a wild uncontrollable torrential flood that tore out the dam and swept headlong to the sea. The fact that longitudinal cracks were found in that block of the dam remaining in place after the disaster may indicate what was the primary cause of the failure

[112] "Tufa Cement, as Manufactured and Used on the Los Angeles Aqueduct," by J. B. Lippincott, Hon. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXVI, December, 1913, p. 520.

or what was a lesser cause. Foundation conditions being such as they were, however, it scarcely seems necessary to seek other causes. At least it is to be said that the block of the dam which remained in place remained despite the longitudinal cracks in it. Whatever the initial cause, it would be well if Mr. Crosby's description of the failure of St. Francis Dam were widely read (see heading "Dams on Sandstone and Conglomerate").

The inspection gallery that parallels the upstream face of Pardee Dam is approached by a gallery extending through the dam from the downstream face. Athwart this approach gallery is a series of longitudinal cracks paralleling the axis of the dam. These are of varying inclination and change from parallelism to the downstream face near that face to practically a vertical direction near the upstream face. On a number of occasions the writer had heard these referred to in rather sober, serious tones. It was somewhat surprising, then, on visiting the dam and being shown these cracks, that considerable difficulty was experienced in locating them. The approximate position of the first crack was known to the guide but it seemed impossible to find it in the darkness with a flashlight. Finally, however, a blue keel mark was found on the wall and beside it a fine crack, possibly $\frac{1}{64}$ in. wide, which in a quite irregular course encircled the gallery. A repetition of this process of search and discovery at 30-ft or 40-ft intervals located the entire family of cracks, all of which, as now remembered, were dry. Although their varying directions were of interest, the writer was at a loss to understand what there was about them to give them a rating higher than insignificant. If the rock abutments and foundations supporting the dam contained cracks and fissures no wider or more closely spaced than these cracks they would be acclaimed as of outstanding excellence even though the jointing lacked anything corresponding to aggregate interlock which characterizes the crack in concrete.

This same aggregate interlock across cracks, particularly across cracks of smaller breadth, is not without points of distinct superiority to the smooth formed surfaces of keyed block jointing even though the latter is grouted. Mr. Steele's comments, under "Slots Versus Grouted Joints Versus Open Joints," indicate that grouting is not, by the fact itself, automatically successful for sealing joints. With aggregate interlock and without separation of parts it is impossible to have movement or slippage or any consequential magnitude along the general plane of a crack without complete shearing of the projecting aggregate, shattering of the adjoining concrete, etc. Without such concomitant manifestations the reported cases of movements and offsets at cracks of necessity must be slippages on pour-planes, unkeyed vertical joints, or similarly formed and created separations resulting from construction processes.

The point of the preceding discussion is in nowise to disparage standard practise, but definitely to advance the idea that small cracks developing in concrete dams are not the desperately dangerous things which so much current practise and discussion, either by implication or direct statement, indicate them to be. If it has been assumed that there will be no crack, then assuredly a crack does violence and affront to the assumption; but the existence of a small crack does not mean that stresses cannot and will not be transmitted across it. Neither does it mean, from the recorded behavior of dams containing cracks,

so far as the writer is aware, that seriously harmful consequences will ensue from them. In short, their hazard appears to be more an assumed one than an actual one. Therefore, although cracks are, in general, to be minimized and avoided, if small ones do occur they can be contemplated with equanimity. They may impair the "structural integrity" of the dam slightly but the actual damage and harm they cause is very slight indeed.

Present practise in the construction of gravity dams is to provide durability, impermeability, and continuity throughout the entire body of the dam—that is, the same high quality of concrete is used in all parts, block construction with its forms and keyways and water-stopping and grouting are used, lifts are made shallow with time intervals for heat radiation and with careful cleaning of surfaces, and the concrete is cooled by circulating water through embedded pipes. By these measures a uniform quality of concrete and a uniform high degree of durability and impermeability are provided throughout the entire structure. There can be no question as the the splendid results achieved by these carefully executed methods and processes, and to question the sufficiency either as to structure or foundations of any of the great modern dams built of proved materials is indeed to utter idle words.

Nevertheless, as Mr. Steele observes (latter part of the section on "Joint Spacing"), "All of these factors affect the cost of the finished concrete * * *." They do that, of and by themselves, and to the further extent that it is wholly impossible with these numerous and time-consuming processes for work to proceed at other than a corresponding rate. Any one who has had opportunity to observe a concrete gravity dam under construction and has followed this with observation of a rolled-fill earth dam under construction, cannot fail to be impressed by the restriction and complexity of processes of the former, by the ease and simplicity of the latter, and by the obvious contrasting costliness of the two operations.

Consideration over a period of time leads the writer to believe that with respect to lowered cost and also in adaptability to foundations that might appear questionable for the present standard dam there is a field of usefulness for what may be called a "concrete fill" dam; that is, a heavy gravity section constructed of lean concrete to be built from the bottom up in a single mass, layer upon layer, in the same manner that rolled-fill dams are built, with impermeability and durability provided at the surfaces of exposure. Essentially what is contemplated is a gravity dam which by a large increase in mass and cross section would permit a substantial reduction in unit cement content and a great simplification of construction processes. As an average case (see Fig. 28), let it be assumed that the upstream face has a slope of 10 on 1, and the downstream face a slope of 1 on 1. Forms would be used on both upstream and downstream faces, but unless conditions of stream diversion necessitated otherwise there would be no joints from abutment to abutment, and the only restrictions to a contractor's free operations within the main boundary limits would be those imposed by the construction of inspection galleries, conduits, and similar elements.

For the main body of the dam a lean, damp, almost dry concrete would be used—0.50 to 0.60 bbl cement per cu yd supplemented with fines. This would

be spread in thin layers and vibrated by bulldozers operating over the top surface of the dam as it was built up, sprinkling and water curing following as the work advanced over successive areas. By its low cement content such concrete, developing 2,000-lb to 2,500-lb strength in a few months, would automatically generate but one half to two thirds the heat of ordinary dam concrete and would have a correspondingly reduced tendency to volume change from that cause. Its modulus of elasticity likewise would be low, and therefore such concrete would the better accommodate itself to slight settlements and adjustments of foundations. If subjected to abrasion and erosion for short periods it would be adequately resistant, but not, of course, for months or years.

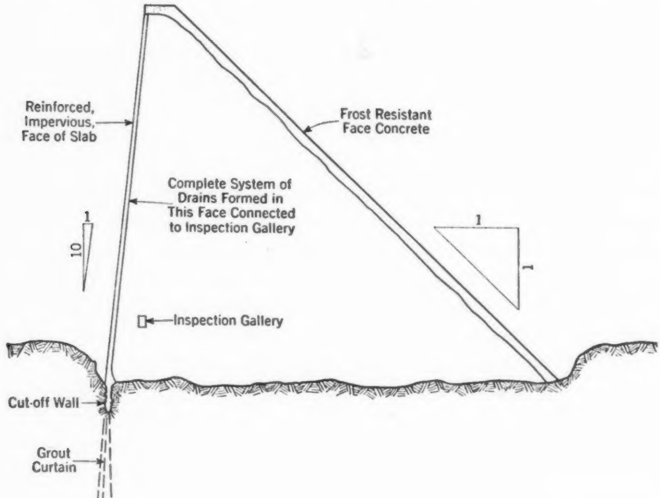

FIG. 28.—TYPICAL CROSS SECTION, CONCRETE FILL DAM

As stated, impermeability and durability would be provided at the exposed surfaces. For the downstream face a zone or layer of richer weather-resistant concrete integral with the main mass would be provided, as has been done in numerous instances in the past. At the upstream face would be located the water-stopping for the entire dam.

Two dams, both of which have their water-stopping (their impermeability) provided at the upstream face, have been described in the technical press.[113, 114] One of these is the repaired Ringedal Dam in Norway; the other is a steel-faced rock-fill dam near Colorado Springs, Colo. Both of these structures have impervious facings which transmit water pressures but at the same time cut off the penetration and passage of water to the main bodies of the dams. Quite similarly with the "concrete-fill" dam there would be provided a reinforced facing slab of suitably rich, impermeable concrete. This would lie directly against the main dam but would be separate from it, the contact surface being formed

[113] *Engineering News-Record*, October 27, 1932, p. 498.
[114] *Civil Engineering*, January, 1939, p. 7.

as smooth as possible and asphaltic coated to minimize friction. For holding the facing slab to the main dam and affording lateral support against buckling under its own weight as a long column, anchors embedded both in the face slab and the dam would be provided, the connection being suitably detailed to permit slight vertical movements between the two. The face slab would connect with a heavy cutoff wall also separate from the body of the dam, which would be complemented by thorough grouting. Formed in the face of the main body of the dam at the contact surface would be a complete set of channels for collecting any leakage that might develop. These channels would outlet into drainage-inspection galleries in the dam. Preparation of foundations and control of concrete would conform with current practise, although the more rigorous details of concrete control would not be necessary for the main dam. However, there would be a complete elimination of block jointing, grouting, pour-plane cleanup, cooling, and similar present processes. The fullest opportunity for speed and freedom of operations would be afforded.

Such a dam, possessed of a certain measure of flexibility, could well be used at sites deemed unsuitable for the standard gravity dam or arch. In its lowered cement content, elimination of block jointing and forming and of the other enumerated steps and operations, and in its reduction in the cost of placing concrete, it would effect large savings. The writer is of the opinion that, where foundations are suitable, it would prove more economical, in the greater heights, than rolled-fill dams.

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DISCUSSIONS

RECOMMENDED PRACTICE AND STANDARD SPECIFICATIONS FOR CONCRETE AND REINFORCED CONCRETE

Discussion

By Edward C. Gould, Assoc. M. Am. Soc. C. E.

Edward C. Gould,[40] Assoc. M. Am. Soc. C. E. (by letter).[40a]—The Joint Committee Report has many of the characteristics of its predecessors which make it difficult, if not impossible, to design without some violations. Most concrete design is for minor buildings, designed under conditions that do not allow long and intricate analysis. The owners want these buildings almost immediately after deciding to build them. The cost of the materials for an inefficient design is forgotten in all the hurry, and is too small an item to balance against a day or two of delay.

By various methods of coefficients, constants, stiffnesses, and approximations, an experienced engineer can, in a reasonable length of time, arrive at a satisfactory idea of the shape and maximum values of the shear and moment diagrams for a frame. By the time this man proportions the reinforcement and size of the members, with due regard to moment, bond, anchorage, and shears, and arranges the steel so that it can be placed in the forms by an iron worker, the owner will hire another designer who will "bat" out a design, and the building will be occupied.

The design of columns after the determination of moments is a very difficult problem. If spiral columns were designed for the axial load to be carried by the core (which is not entirely correct), and if the covering were allowed to take the moment, then up to a certain value of $\frac{e}{t}$ the eccentricity could be neglected. Beyond this point, tied columns should be used, with the steel placed at the faces where it works to best advantage. For higher values of $\frac{e}{t}$, moments are not carried efficiently by spirals in any case.

Note.—This Report was published in June, 1940, *Proceedings*, Part 2. Discussion on this Report has appeared in *Proceedings*, as follows: September, 1940, by L. J. Mensch, M. Am. Soc. C. E.; and November, 1940, by Messrs. John C. Sprague, and Walter R. Hnot.

[40] Senior Architectural Engr., Bldg. Dept., City of Minneapolis, Minneapolis, Minn.

[40a] Received by the Secretary October 23, 1940.

Another possibility is an attempt to eliminate bending in columns. This could be done to a certain extent by placing all of the moment steel in the bottom of the beams. Instead of using heavy steel for negative moment, light bars or mesh can be used over the supports to spread the deflection cracks. This expedient, of course, would stress the material beyond its elastic limit. This simple beam design, most certainly, should be used for the first and second floors of all general-purpose buildings such as stores and offices where alterations must be made frequently during the life of the building. Every one has seen "beautiful" cases of continuity destroyed by the installation of a stair, ramp, or service elevator.

In all cities there are old buildings that have some cracking over supports and are probably now acting as simple spans. They are still good buildings with considerable carrying capacity. If these unsightly deflection cracks are spread out so as not to be visible, simple beam design is good.

There is evidence of yielding in concrete both by test and observation of buildings of some age. Whenever compression steel in beams is required, it should be designed at the same value as tension steel, and should be designed for the entire compression. This apparently is the eventual action of the steel. In all columns where the ratio of the dead load to the total load exceeds five tenths, the dead load divided by the area of vertical steel should never exceed the elastic limit of the steel. Even with this provision, every masonry wall in the plane of the columns should have pressure-relieving joints.

In continuous beams it is customary to place some steel in the bottom of the beam at the face of the support. As the concrete yields, this steel will assume more and more compression; eventually it will probably take all if it does not pass the elastic limit. This overstressing of the steel is probably not harmful if it is developed adequately within the support. Inadequate development will overstress the bond on those bars which might slip and will cause an end bearing failure on the concrete, which may or may not be serious.

There are two problems that confront any code committee. The first problem is to provide a standard of safe proportions by which a design may be made by simple moment and shear calculations, or axial loadings. The second problem is to encourage rigorous analysis by allowing considerable economies of materials over the simpler analysis. The first standard should be safe and should have an adequate if not uniform factor of safety. The moments should be determined by the $\frac{W\,L}{K}$ method if the spans do not vary over 20%, and moment should be neglected in the columns. A working stress based on about $0.35 f_c'$ for concrete and 18,000 lb per sq in. for steel should be used.

The second standard should have a uniform factor of safety, possibly smaller than the other standard, and should be used only where elastic computations have been made to find the distribution of moments through the joints at the far end of adjacent members. Moment, shear, and bond should be computed for all critical points, and all combinations of loadings. For working stresses there should be a code similar to that of the Joint Committee,

with modifications in anchorage provisions and simplification of column bending. The writer does not believe that, with plastic flow and other factors that cannot be rationalized completely, the designer should become involved in complicated formulas.

The writer realizes that the best refinements of design should be used on all large and unusual structures. He also feels that the engineer can show economies in material by such refinements in all buildings more than sufficient to pay for his time. However, he cannot prorate his time against the occupancy value of the building.

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DISCUSSIONS

MAXIMUM PROBABLE FLOODS ON PENNSYLVANIA STREAMS

Discussion

BY MESSRS. JOSEPH L. BENSON, H. ALDEN FOSTER, AND EDGAR E. FOSTER

JOSEPH L. BENSON,[2] ASSOC. M. AM. SOC. C. E. (by letter).[2a]—By his thorough but concise presentation of the factors and principles involved in the determination of the maximum probable flood, Mr. Ruff has made a distinct and valuable contribution to the field of hydraulic engineering. Experienced engineers will recognize not only the merits of the data presented but will also appreciate the author's recommendation for caution in accepting his formulas, tables, and graphs as the missing pieces of their own specific jig-saw problems. The critical engineer may not accept the author's findings entirely, but he will undoubtedly find, among the research data presented, certain tools that can be utilized in his own work. It appears obvious that the author does not intend for engineers to dissect his paper for rules-of-thumb to be used as a substitute for whatever rule-of-thumb they may have used in the past for solving their own particular problem.

The writer believes additional thought on this subject may be provoked by calling attention to the following comments:

1. Mr. Ruff describes his "maximum probable flood" as one "so large that the chance of its being exceeded is no greater than the hazards normal to all of man's activities." This is a concept rather than a definition. What are the hazards normal to all of man's activities? Utility will be added to the value of the paper if the author, together with other eminent engineers, can furnish examples for which his "maximum probable flood" would be the design flood and, for the benefit of those who work with frequencies, also correlate this flood to frequency of occurrence based on presently available data. What are the conditions or circumstances for which the author's maximum probable flood would be the design flood? This is important because too often research reports have failed to reach their objective or ultimate possibilities, due to the

NOTE.—This paper by Charles F. Ruff, M. Am. Soc. C. E., was published in September, 1940, *Proceedings*.

[2] Associate Engr., Federal Power Comm., Atlanta, Ga.

[2a] Received by the Secretary October 28, 1940.

omission of qualifying or related statements. The writer believes that the maximum probable flood derived from the author's methods and data would be equivalent to the design flood for a spillway structure (either dam, highway, railroad, etc.), failure of which might cause loss of life, suspension or dislocation of business activity, and in addition direct dollar damages of several times the cost of the entire project. Frequency values of this "maximum probable flood" can be determined for certain particular localities by utilizing the flood-frequency curves of those localities. The "maximum probable flood" will be a computed discharge for a particular locality. This computed discharge has a frequency value as determined from the flood-frequency curve. The approximate frequency of the enveloping curves could be ascertained by the foregoing determination for these points which control the location of the enveloping curves.

2. If sufficient agreement could be obtained on the occasion and purpose for which the maximum probable flood would be the design flood, it would be possible to establish a series of relationships between the protection offered by, and the magnitude of, this design (maximum probable) flood and the protection required by, and the magnitude of design floods for, other situations. At a specific location, the design flood for a bridge on a "farm-to-market" dirt road would be different from a large dam or important transcontinental railway bridge. It is obvious, therefore, that the structure itself affects the magnitude of the design flood. It is also believed that sufficient importance has not been given to the effect of the possible downstream damages on the design flood used for spillway structures and flood storage. Most engineers have realized, in a qualitative sense, that less protection is needed where resulting damages would be small if failure should result. From a quantitative standpoint, engineers have found very little in print on this subject and have had to depend on their own judgment. The effect of structure and potential downstream damage on the magnitude of the design flood is appreciable.[3]

3. The author's method for determining the maximum probable flood involves rotation of the axis of the storm, when transposed over the particular drainage area, to give the maximum runoff from the particular storm. Mr. Ruff assumes that the "severe condition of a perfect fit" compensates for the vagaries of nature in causing a larger storm or the effect of a higher than normal runoff coefficient due to a previous storm. The writer believes the compensation should be provided for by means other than rotation of the axis of the storm. Although little data have been published concerning either the propriety or impropriety of rotating the axis of a storm, it appears best to transpose the storm without rotation.[4, 5] The writer believes that the more uniform the pattern of the storm paths, the less basis can be found for rotation of the axis of the storm. Additional data on the meteorology of floods appear necessary and may be available.[6]

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[3] "Recreational Use of Forest Waters," by Clifford Allen Betts, M. Am. Soc. C. E., *Engineering News-Record*, August 29, 1940, Fig. 4, p. 61.

[4] "Measuring the Discharge of Texas Streams," by J. L. Lochridge, M. Am. Soc. C. E., *Civil Engineering*, September, 1937, p. 638.

[5] "The Sardis Dam and Reservoir," by Norman R. Moore, M. Am. Soc. C. E., *Civil Engineering*, June, 1939, p. 349.

[6] "Flood-Protection Data," Progress Report of the Committee, *Proceedings*, Am. Soc. C. E., February, 1938, p. 338.

4. The validity of transposing the isohyetals of a storm to determine the "maximum probable flood" depends on: (a) The probability that the storm would or could center over the drainage area in the future, and (b) reliability of the isohyetals drawn from the recorded rainfall. In the former case engineers have been sufficiently warned concerning the application of air-mass analysis without expert advice.[6] With regard to the latter, there is nothing so spotty as spotty rainfall.[7] There should be a sufficient number of Weather Bureau stations within the area of heavy rainfall not only to delineate the isohyetals properly, but also to permit sufficient reliance to be placed on them. If there is a choice between using the method of transposing isohyetals or using flood runoff data, the writer suggests using both. Of course, the weight to be given to each solution will depend on the reliability of the data used for each method, especially the period of time for which the rainfall and river runoff data are available.

5. The author uses the enveloping curve for the storm rainfall producing the maximum probable flood and provides no factor of safety as a surcharge (at points controlling the location of the enveloping curve) for possible storms in excess of those already recorded. By rotation of the axis of the storm as indicated in the preceding paragraph, the author actually obtains a factor of safety, in the sense that rotation of the axis of a storm will produce a flood several times larger than would be the case without rotation. However, would not the experiences with enveloping curves of flood runoff be repeated with enveloping curves of maximum rainfall? William P. Creager, M. Am. Soc. C. E., believes that if enveloping curves of record storms "* * * were derived and plotted, they would show the same increase with time, that is, an increase with the accumulation of data." [8] It is possible that the author can offer additional information on this point. The enveloping curves of maximum rainfall may be a definite derivative of the pattern of storm intensities, but, when the location and slope of the enveloping curve are determined by only two or three storms, there is a strong possibility that such curve will be pierced at several points as time goes on. This possibility need not detract from the value of the author's work, if it is definitely understood that the enveloping curve is a tool in determining the maximum probable flood and is not of itself a representation of the maximum probable storm.

6. Mr. Ruff's definition of runoff coefficient is admirably suited to his method of determining the probable maximum flood. However, more accuracy can be obtained by converting fallen snow to equivalent inches of rainfall. If this change were adopted, the author's method could still be utilized by adjusting the enveloping curve of the winter storms to include the expected snowfall conditions. Many engineers will find great interest in the emphasis on seasonality and temperature, size of storm, and character of the watershed as affecting the runoff coefficient. Apparently, size of drainage area does not affect the runoff coefficient other than to provide a place for the watershed characteristics to take effect.

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[7] *Proceedings*, Am. Soc. C. E., January, 1940, p. 178.

[8] "Possible and Probable Future Floods," by William P. Craeger, *Civil Engineering*, November, 1939 pp. 668–670.

7. Engineers who have made flood-flow studies based on the same storms used by the author should present their findings. For example, the storm designated as 244, March 12–15, 1929, near Elba, Ala., was used as the basis for determining the spillway capacity of the Sardis Dam in Mississippi. Likewise, the storm designated as 248 may have been used in spillway studies of Saluda Dam in South Carolina. It is logical to assume that, whenever important hydraulic structures have been erected in the locality, adjacent and subsequent to one of the designated storms, the designing engineer would compare his design data with those obtained by using the author's methods. Engineers who have used one or more of the specific storms represented in Tables 2 and 3 could, therefore, contribute comparative data to the discussion.

8. The value inherent in some parts of the author's presentation cannot be overemphasized. So much emphasis has been placed on peak flows that some engineers talk of nothing else, whereas equal or greater emphasis should be placed on flood volume or flood volume above a specified rate of flow. Fig. 14 and Table 8 illustrate both peak flow and flood volume. Peak flows have utility as: (*a*) An index of flood magnitude and hydrograph construction; (*b*) a criterion for spillway design of run-of-river hydro-plants or other structures where no flood storage is used in the design; and (*c*) development of backwater curves. Since most reservoirs provide an appreciable flood-storage capacity (usually that storage between the maximum and normal water levels), the importance of flood volume cannot be overemphasized. In the case of most flood-control reservoirs, it is flood volume above a stated rate of outflow, usually limited to non-overflow or nominal overflow of the river channel. Where downstream conditions do not limit the outflow for structures, it is flood volume above the average spillway capacity. It is realized that practically all engineers take flood volume into consideration when that item affects the design of the spillway structure; but further emphasis on flood volume above the relevant flow is desired in technical publications and discussions. The relative importance of peak flow and flood volume, of course, is dependent on the locality and the design of the structure. The difference between inflow and outflow is obviously the difference in the volume of water going into or out of storage. One extreme is the run-of-river hydro-plant with little or no storage where outflow or spillway capacity must approach or equal the inflow, and the other common extreme is the normally empty flood-control reservoir where outflow may be held to zero or a very small percentage of inflow.

9. The author's admonition to those tempted to use the published data for their own studies can be most appreciated by those familiar with the vagaries of nature in bestowing heavy rains at unexpected places and times. When combined with other peculiarities, floods result which can scarcely be predicted or provided for with statistical logic. For example, the designer of a spillway structure in the northern part of the United States must contend with fallen snow and ice; in the southeast the twin-peak flood gives emphasis to flood volume rather than peak flow; in Texas and other parts of the southwest, the 1935 floods show peak flows considered by some engineers to be "a phenomenon of a very special nature"; in California and the far west it is reported that peak flows have included a transient peak containing 50% water and 50% solids.[9]

[9] "Transient Flood Peaks," by Henry B. Lynch, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., November, 1939, p. 1623.

Engineers who must work with the more peculiar storms and watersheds will recognize the necessity for good judgment in determining their own design floods.

The writer's comments are intended to emphasize the author's own warnings concerning the use of data prepared for particular purposes. In almost every instance the provision of floodway capacity and flood storage is a matter involving dollars and sense. The engineer is guided, consciously or unconsciously, by the desire to make provision for expected and unexpected floods to the extent that increasing increments of flood protection do not require unreasonable outlays of money. If additional flood protection can be obtained at little or no cost, the engineer will be more than generous in providing such protection. If such additional expenditures are uneconomic from the financial and social viewpoints, the engineer will be tempted to restrain his imagination. The substance of the matter can be summed up as follows: Always provide sufficient protection for all reasonable contingencies and then add whatever additional protection that can be obtained without unreasonable added expense. Consciously or unconsciously, in matters not subject to mathematical precision, the engineer must be guided by dollars and sense.

H. Alden Foster,[10] M. Am. Soc. C. E. (by letter).[10a]—For the last twenty years (1920–1940) the writer has followed with interest the various methods proposed for estimating stream flow in the design of reservoirs and dams. There has been a noticeable trend in the development of these methods, which may be worth considering. In 1914, the late Allen Hazen,[11] M. Am. Soc. C. E., made a revolutionary step in studies of hydrology by proposing the probability method for analysis of stream flow. This method received much attention from members of the profession and was extended in various directions.

Previous to the publication of Hazen's original paper, the late Weston E. Fuller,[12] M. Am. Soc. C. E., had introduced the idea of probability or frequency in the study of floods.

There have been numerous variations of Hazen's and Fuller's ideas, both in estimating future stream flow and determining maximum floods for design of structures. Fundamentally, however, all these methods were based on the assumption that, if a continuous record of stream flow or flood intensities were available covering a number of years, it could be used as a basis for estimating the probability of occurrence of a rate of flow or flood intensity of any given magnitude in the future.

The chief difficulty with the probability methods was that, in most cases, the available records were not sufficiently long to serve as a basis for reliable estimates of probability for periods exceeding the length of the available records to any great extent. Thus a probability curve based on a 10-yr record might be extrapolated to represent a 15-yr or 20-yr record; but it could not safely be extended to estimate the results of a 100-yr record. As few complete stream-flow records were available for periods of more than twenty to thirty years, the method did not seem to be of much use in some branches of hydraulic design, such as for flood control and particularly spillways of dams,

[10] With Parsons, Klapp, Brinckerhoff & Douglas, New York, N. Y.

[10a] Received by the Secretary October 28, 1940.

[11] *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 1539.

[12] *Loc. cit.*, p. 564.

where it was generally felt that the designs should be safe against floods whose probability of occurrence was less than once in one thousand years or more.

With a realization of this limitation of the probability method, efforts were made to develop a method for deriving a hydrograph of the "maximum flood" or the "maximum probable flood" which could be expected to occur on a river in a given location. Various ingenious methods were developed for this purpose. It is not the purpose of the writer to review these various methods, but only to indicate their trend. In recent years, the tendency has been to develop a synthetic hydrograph, based on a synthetic rainfall record. The rainfall record is supposed to represent that of the maximum storm which could occur over the watershed, or a storm of an intensity that would be expected at only rare intervals. In some of the methods, the principle of probability was used in deriving this maximum storm; in other methods (such as those described in the paper) the maximum storm is assumed to be greater than any on record but no attempt is made to estimate its probability of occurrence.

Now it happens, in the United States at least, that rainfall records are available covering much longer periods of time than records of stream flow or of floods. It is practical, therefore, to apply the probability methods to rainfall records in many areas where a similar application to stream-flow studies would be quite unreliable. Such a combination of probability methods applied to precipitation studies with synthetic runoff records has been adopted by the U. S. Army Engineers, particularly in arid regions where stream-flow records are quite inadequate.

In the paper, the author has developed a very ingenious method for obtaining a "maximum probable storm," which he considers to be of sufficient rarity to deserve this description, but without attempting to estimate the probability of its occurrence in any year. By analysis of existing stream-flow records in the area under consideration, he develops a method for obtaining the 100% runoff hydrograph produced by this storm on a "standard watershed" of an assumed shape and size. To apply the method to any particular watershed he uses the following procedure:

(1) Prepare the storm hydrograph for the maximum probable flood on the standard watershed, corresponding to the actual drainage area of the watershed under consideration and the particular season of the year;

(2) Multiply the standard flood by a "location correction factor," which is supposed to correct for the variation of storm intensity at changing locations in the state;

(3) Multiply again by a "shape correction factor," which is supposed to adjust for the fact that the given watershed has a shape different from that assumed for the standard watershed; and

(4) Multiply again by a runoff coefficient, to make allowance for the fact that the actual runoff percentage will not be 100% of the rainfall, as assumed in preparing the standard hydrograph.

The final result will be a synthetic hydrograph of runoff in cubic feet per second per square mile; when this is multiplied by the actual drainage area in square miles, the final desired hydrograph will be obtained.

It will be observed that each of steps (1) to (4) involves the use of a quantity or factor which, in itself, is of uncertain accuracy. For example, if each step involved a positive error of 10%, the over-all error would be 47%. Even if the several steps should have margins of error considerably less than 10%, the final result might still be considerably in error.

The point that the writer wishes to emphasize is that any method which involves so many assumptions in its various steps cannot be expected to produce very reliable results; and since the final results cannot be considered precise by any means, the question may be asked whether equally good results might not be obtained by some less complicated method. The writer would suggest for such a simplified method that a probability analysis be made of the rainfall records to determine the ratio in magnitude and intensity of the "1,000-yr" storm (for example) to any given storm. Then the hydrograph of the flood produced by the given storm would be adjusted to obtain the runoff corresponding to the 1,000-yr storm.

The method used by the author for determining the variation of storm magnitude with geographical location is very interesting, and so far as the writer knows this method has not been described previously in print. The basic assumptions under which this method is developed appear to be sound, and very likely could be applied to advantage in other localities.

It is possible that the writer's impression of the complexity of the method would not be fully substantiated after practical application; familiarity often breeds respect. The paper is well worth study for its explanation of the various factors that influence the size of floods; and the writer is to be congratulated on the clear yet complete manner in which he has presented his subject.

Edgar E. Foster,[13] Assoc. M. Am. Soc. C. E. (by letter).[13a]—In this excellent paper the author has presented a method of devising synthetic design floods that is based essentially upon statistical analysis. The method is statistical in that the results are based on averages or extreme variations of a large number of observational data of the principal factors entering into the production of a flood. The resulting design floods, therefore, are not derived from one or, at most, a few great storms.

It appears to the writer that statistical analysis should constitute a sound basis on which to construct design storms, for the reason that hydrologic data are primarily statistics of the various elements of storms and floods. The method should be particularly adaptable where many design floods are required in a climatologically homogeneous region.

Storms and floods are complex phenomena, and in order to gain an adequate insight into them, they should be studied piecemeal. The primary elements of a storm are the rainfall, the duration, the area covered, and the location. Of these four elements, three are commonly expressed as numbers and therefore can be treated by statistical methods. Because of the great variation of the data, they can be conveniently expressed as averages and maxima. The fourth element, location, cannot so readily be expressed numerically, but it is the result of movements of the interacting air masses and depends largely upon

[13] Associate Engr., U. S. Engr. Office, Omaha, Nebr.

[13a] Received by the Secretary November 4, 1940.

the variation in atmospheric pressure and directions and velocity of the wind. Behind these four elements there are other more remote factors such as moisture content of the air and difference in temperature between the air masses, which play a part in the production of the storm and resulting flood. These factors may likewise be expressed as arithmetical data and are variable quantities, although the effects are not so readily traceable.

Still another element is necessary for the production of a flood even when heavy rainfall is received—that is, favorable ground conditions. This element is expressed as a numerical coefficient, being a ratio of the runoff and the rainfall, or as a rate of infiltration.

All these numerical data may be treated as statistics so that storms and floods can be analyzed into their constituent parts and averages obtained or envelope curves be drawn over the greatest observations to obtain probable maximum values. Since floods deal with maximum values, envelope curves are particularly valuable as shown in Figs. 2, 5, 7, 8, and 17. These averages and maxima can thereupon by synthesized into a design flood. This procedure is essentially the one the author has followed and is, in the opinion of the writer, a sound and logical process.

The mechanics of the method probably will cause more disagreement among engineers. However, the same statement can be made of any other series of steps in developing design floods, for the reason that the art is as yet too young to have rigidly established procedures.

This situation applies particularly to the standard drainage areas as used by the author. This method of obtaining the distribution of runoff is empirical to a high degree and does not have a statistical basis comparable to that of the data of rainfall or other elements of storms. Like other formulas of similar nature, its use should be limited to the area and conditions under which it was derived. Furthermore, the derivation omits the effect of slopes and valley storage, elements which have an important bearing on the distribution of storm runoff.

Because there are a number of features in the paper that could be explained or harmonized by an understanding of the part played by the various air masses in the production of floods, it is regretted that the author did not include a more extended discussion of them.

It is desirable to trace the paths of the storms—that is, the movements of the areas of low barometric pressure, constituting the center of the extratropical cyclones. However, these areas of low pressure do not tell the entire story in themselves. The principal function of these "lows" is that they constitute a mixing pot for the various air masses in which the warm moist air is more or less rapidly lifted over the cold air so that the moisture content is precipitated with great intensity. Whether heavy precipitation occurs in the "low" depends upon the types of air being drawn or perhaps pushed into the center. For example, the "low" of the storm of March 16 to 18, 1936, entered the west coast of the United States on March 10 and produced little or no rainfall until March 16 at which time it was located over Louisiana. From that time on, it encountered the tropical air masses and heavy rainfall occurred. It should be noted in passing that the heavy rainfall which partly caused the heavy floods in New England occurred while the low was over the Middle Atlantic States; in

this case the heavy rainfall cannot be accounted for except by movement of the tropical air masses over New England.

East of the Rocky Mountains, there are two types of air masses that are important in the production of heavy rainfall such as is required to cause a flood; these are the Polar Continental type (being named for the region of their origin) characterized by cold dry air, and the Tropical Maritime type that is the warm moist air of the Gulf of Mexico and adjacent tropical seas.

The latter type is the carrier of the moist air and the producer of the heavy rainfall necessary for floods east of the Rockies. The farther they travel from the source region, the more likely it is that the moisture content is depleted and the more likely that the supply of moist air will be interrupted or cut off by movement of the cold and dry polar air. In the United States this condition will not have a noticeable effect on the short storms such as thunder-storms during which moist air will be concentrated by convective action so that rainfall with a duration of a few hours may be nearly as intense in the northern states as in the southern. The great storms arising from air-mass action alone, however, require a continual supply of moist air to produce the precipitation observed, since the moisture content in a vertical column of air is not enough to produce the rainfall observed. As the tropical air masses move northward they leave their source of supply and at the same time approach the region of the cold air masses, which, being more vigorous in the northern latitudes, more readily cut off the supply of moist air. This results in a decreased total precipitation as the storms are located northward.

Figs. 1, 3, and 4 show the effect of the interrupted and diminished supply of moist air. Fig. 1 shows the steady decrease in rainfall along the axis of the Atlantic Coast of both summer and winter storms. The decrease appears (see Fig. 2) to be somewhat greater for the latter as may be expected because of the colder water surface that must be traversed from the source region of the tropical maritime air masses.

Likewise the storms moving northward from the Gulf of Mexico, shown in Fig. 5, have a diminishing total precipitation. This decrease is also shown by Figs. 3 and 4, along the southeastern extension of the axis. The peak (about mile 1300 on the Ohio Axis for summer storms) being located over the eastern border of Pennsylvania and New York, may be caused by the tropical air masses that come from the South Atlantic and enter land in the Middle Atlantic States. The dip in the axis in Fig. 4, between mile 800 and 900, remains unexplained unless the Gulf air loses its moisture before it moves eastward along the Ohio River Valley. Perhaps the storms are not susceptible of being sorted arbitrarily into summer and winter classes, or it may be that too few storms have been experienced to permit a certain classification. There seems to be no very good reason why winter storms should yield greater precipitation in western Pennsylvania than do the summer storms.

In any case the source of moisture is the same for both winter and summer storms, which fact accounts for the similarity in the two types of storms as noted by the author. There are some modifying influences, most important of which is the lower temperature of the gound that must be traversed in winter; this condition results in a smaller moisture content by the time the air masses reach the storm region.

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DISCUSSIONS

THE GRAND CENTRAL TERMINAL IN PERSPECTIVE

Discussion

By Messrs. F. Lavis, E. R. Hill, Alonzo J. Hammond, and H. L. Ripley

F. Lavis,[22] M. Am. Soc. C. E. (by letter).[22a]—This all too brief review of the events and processes leading up to the final development of the present Grand Central Terminal in New York is a paper of the kind that engineers need so much and so seldom see. It is an account of particular interest to all who look beyond surveys and the calculation of the size of an I-beam, or even of an unprecedented bridge or other structure, to the fundamental effect of engineering structures and engineering design on economics and on public and civic affairs. The author is to be congratulated on having written and on being able to write such a paper.

One hesitates to criticize it, therefore, but knowing the author for what he is, a passionate servant of the truth, the writer believes that he will accept a very slight amendment to his statement about the effect of this improvement on assessed valuations in Westchester County.

That the terminal improvement and the electrification of the railways within the northern suburban area of New York City have contributed to the building up of Westchester County no one will deny; but it is to be questioned if this expansion can be attributed to this improvement to the extent indicated in the paper. Assessed valuations in Westchester were increased, materially and arbitrarily, in 1920 and 1921. In Scarsdale, for instance, assessed valuations were doubled at that time, from about $10,000,000 to $20,000,000, with only a very small increase in actual values. Similar increases occurred throughout the county at about the same time in an attempt to comply with the law which requires assessments to be equal, as nearly as possible, to the full value of the property.

In attempting to show that the rate of increase in Westchester County is very much greater than the rate of increase in the state as a whole, it must be

Note.—This paper by William J. Wilgus, Hon. M. Am. Soc. C. E., was published in October, 1940, *Proceedings*.

[22] Cons. Engr., New York, N. Y.

[22a] Received by the Secretary October 29, 1940.

borne in mind that Westchester is the nearest available suburban area to New York, the largest and richest city in the world. Long Island and New Jersey are separated from Manhattan by river barriers, whereas Westchester has easily available land connections and the railways serving it have a terminal in the heart of New York City which, during the period considered by the author, was additionally connected to downtown New York by the first subway line. The growth of Westchester has been coincident with the growth of New York City, both in wealth and the number of inhabitants, and must be attributed in part to this growth of the city.

During the period under consideration, Westchester was also building the Bronx River Parkway, the first of that series of great highway arteries which have made the county so easily accessible; and as a matter of fact the sponsors of this parkway have claimed that it was the large expenditure of county money for this purpose that has been responsible for the great rise in land values and assessments in the county. Certainly these parkways are responsible for helping to make Westchester a desirable place in which to live.

In spite of these facts, however, the author has ample claims on the gratitude of the profession in describing a remarkable piece of work very well done. From his own experience, the writer can testify to the competence of the organization which did this, handling millions of passengers while the work was being completed with scarcely any delay or inconvenience.

So far as Westchester is concerned, even though it may not be conceded that its growth was wholly due to the terminal improvement and the electrification, it must be conceded that the two went hand in hand—that a great public utility had vision and measured up to its duties and responsibilities.

E. R. Hill,[23] M. Am. Soc. C. E. (by letter).[23a]—The traveling public takes for granted today the two major passenger stations of the railroad on Manhattan Island. The generation is passing that saw the building of the Pennsylvania Station and of the present Grand Central Terminal, and it is therefore timely and fitting that the full story of the latter should be spread before the members of the Society.

This station presented many problems differing from those connected with the Pennsylvania Station. This latter was an entirely new project involving tunnels under the Hudson River from New Jersey and a connection by tunnels under the East River with a large new train yard on Long Island. The station, therefore, partook more of the nature of a major stop en route than of a stub-end terminal.

The problem before the New York Central Railroad Company, however, was the rebuilding and expansion of a busy operating terminal station, together with the reclaiming of valuable building sites by the placing of the tracks underground. This involved the adoption of electricity as the means of motive power at a time when electric traction was still in its formative stage.

The difficulties of such an undertaking were enormous, of course, for, besides the physical problems encountered, it must be remembered that all of

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[23] Cons. Engr. (Gibbs & Hill, Inc.), New York, N. Y.

[23a] Received by the Secretary October 30, 1940.

this work had to be done without interference with the operation of the railroad with its complex combination of through passenger service from the north and west and, for that day, a heavy commuter service. Some of the older commuters will remember the wooden ramps which were built at various degrees of slope to test out just how steep they could be made without slowing up the march of the crowds unloaded from the trains.

The transformation of Park Avenue, then ill-named, from its smoky and generally undesirable character to its present state, also is one of the outstanding features of the project, taken very lightly and casually by the average traveler of today.

That the creators of the terminal built well is indorsed by the fact that, notwithstanding the enormous growth of the traffic handled, the station has continued to do its work satisfactorily with virtually no major changes. The well-planned separation of the through and commuter traffic remains as it was at the outset and handles the tremendous crowds of rush hours and holidays with an efficiency that is the admiration of all visiting railroad operators. Mr. Wilgus' data, given in Table 4 and supporting text, are impressive, the percentage of increase in traffic being 264 for commuters and 166 for all kinds of traffic. These are yearly totals; it is probable that the daily peaks would show even greater increases.

Mr. Wilgus points out that the time approaches when thought must be given to handling a further increase of traffic. Who shall say what the future holds? Are cities in the United States to grow indefinitely or are they to feel increasingly the effect of decentralization of which there are signs? Whatever is ahead, one can but hope that men will be found of the caliber and character of their predecessors to solve the new problems as they arise.

ALONZO J. HAMMOND,[24] PAST-PRESIDENT, AM. SOC. C. E. (by letter).[24a]—In the "Synopsis" of this paper, the author rightly refers to the exceptional advantages achieved in the interest of the owner, and of the public as well, by the utilization of overhead air rights due to the change of motive power from steam to electricity.

The fact that the approach tracks ran through a public street where the steam and smoke were objectionable, as well as in the zone of the terminal proper, made it all the more important to change the motive power—the result of which, of course, was "a clean bill" for the very remarkable development of air rights. If the terminal alone had been considered, the result could have been obtained, as it has been in the case of the Chicago Union Station, Chicago, Ill., where steam is the motive power.

During the earlier stages of construction of the Chicago Union Station, full-scale tests were made over quite a period of time to determine the best type of inlet areas, and the type of fan and power required for removing promptly steam and smoke from a locomotive through ducts to a satisfactory outlet. The results obtained made possible the use of air rights for the Daily News Building and the Post Office, the latter extending for a block about 800 ft long

[24] Cons. Engr., Chicago, Ill.

[24a] Received by the Secretary November 18, 1940.

and the full width of the station tracks. It will be admitted that the results do not measure up to those obtained with electric traction, but are fairly satisfactory, with one important exception—the corroding effect of smoke.

The author has been able to add a new term to the classic definition of engineering, and he is to be congratulated not only in having the vision but the opportunity to "evoke" such a magnificent station development. In consequence, he has well earned the retrospect of a job well done.

In connection with his studies for the Chicago Union Station, the writer had occasion to spend quite some time making an intensive study of the Grand Central Terminal, the engineering design, operation, income from facilities, economics of investment, as well as architectural treatment for combined station use and office facilities. The writer was impressed most by the intensive use of vertical space in a two-stage development and the minimum effort required of the passenger to reach the various places he had to go to supply his wants for service—the ticket offices, parcel room, information booth, entrances to the train concourse—all in plain sight and easy of access when he reaches the foot of an easy ramp from the street, or a few short steps after leaving his taxicab. Such a well-planned sequence of movement for the passenger exists in no other station, to the writer's knowledge. Opportunity was there and was taken advantage of to a superlative degree.

The author has given a very enlightening story of the processes of the Electric Traction Commission in reaching conclusions, not only as to type, but extent of electrification, as, for example, to stop at Mott Haven, five miles out, or go to Harmon, about thirty miles farther. Table 4 shows the increase in commuters from 1906 to 1930, but it would have been of interest if the immediate effect of electrification could have been noted resulting from greater speed and comfort of the suburban trains. This is mentioned for the especial reason that beyond a certain zone of the Illinois Central suburban electrification in Chicago, both on the South Chicago branch and the main line, there was a very prompt reaction in building, and new neighborhoods developed, increasing the local traffic quite promptly.

The architectural compromises referred to and the statement that the intended high office building over the south end of the station on 42d Street "fell by the wayside," lead one to the conclusion that the ultimate result was most fortunate in providing an elevation on 42d Street, facing south into Park Avenue, of matchless beauty and, in addition, the glassed arched gables permit the sun's rays to pour down into the passenger concourse with the result that the people are kept in a cheerful mood when waiting for a train.

Such a development requires many studies to be made and many discarded. In the case of the head house of the Chicago Union Station, there were some thirty studies made. The caissons were put in for the selected type before the United States entered the World War, and afterward a different design was adopted, necessitating a revision of nearly all the caissons. In the case of the Grand Central Depot, it took courage and vision to tear down the old structures and start "from scratch."

Where there is so much to commend, there is left very little to criticize in such a terminal. The writer has found one element, however, which is inherent

with a stub-end terminal with passenger platform used for handling baggage and mail. The writer has timed the unloading of baggage and mail onto trucks from an inbound train and has found it to be from 20 to 22 min. In the meantime, the passengers have all gone by on the narrow lane left by the trucks; so, without invidious comparisons, one might refer to the Chicago Union Station's separate platforms for such traffic, a detail believed to be peculiar to that station.

At times there have been criticisms from some sources of the money which railroads have spent for what were termed "monumental" stations. In 1916, when Robert Trimble was preparing his presidential address for the American Railway Engineering Association, the writer gave him an editorial comment by John Henry Zuver in a South Bend (Ind.) newspaper, which ran as follows:

> "In the middle ages, communities built magnificent cathedrals—and paid for them—and who questions the ennobling effects of these rich and spacious structures on all beholders? They satisfied and glorified the lives not only of those who built them, but also of their descendants. The genius of the American people runs to transportation and to business. Why not express through those mediums the artistic soul of the nation? Who that ever stood before the massive pillars of the Northwestern in Chicago, who that ever walked the floor of the wasteful concourse in the Pennsylvania Station in New York, with its noble space and simplicity, its astonishing and grateful silence, can ever think of them without a thrill?"

H. L. Ripley,[25] M. Am. Soc. C. E. (by letter).[25a]—In writing the history of the development of railroad service for passenger transportation northerly from New York City, along the Hudson River and into Westchester County and New England, Colonel Wilgus has again done a notable service to the engineering profession and to the public at large.

His description of the peregrinations of the terminal from City Hall Square to the present location of the Grand Central Terminal at 42d Street and Park Avenue would be a worth-while endeavor of itself. His description of the terminal project, however, makes it of vastly more interest and importance to the engineering profession. His description of the terminal and its northerly approach, with maps, tabulations, and photographs, presents the problem in such clear and complete detail that one who has never seen it can form a clear picture of the magnitude of the enterprise as a whole.

He has touched only lightly upon a phase of the matter which, to the writer at least, seems to be the most remarkable part of the undertaking (excepting its original concept), namely, that the entire job was completed under a traffic maintained on the same location, reaching more than 500 regular scheduled trains, and which involved as many as 1,000 separate movements per day.

All engineers know that it is one thing to construct a project "in the open." It is quite another to do the same job in the heart of a great city, under intensive traffic, and to keep that traffic moving without delay or serious inconveniences; and those who had occasion to use the terminal during the construction period know that this was accomplished remarkably well. The excavation had to be

[25] Contract Agt., N. Y. N. H. & H. R. R., New Haven, Conn.

[25a] Received by the Secretary November 22, 1940.

made over acres of area down to a depth of approximately 45 ft below 42d Street, still keeping the trains moving "on time."

As the author states, the station and its environment can now be seen "in perspective." The traveler is likely to overlook the influence it has had in adding to the "more abundant life" of the community and to forget the troubles and tribulations involved in its accomplishment unless he remembers the conditions that existed thirty or more years ago, when the two sides of the city were cut apart from 42d Street to 56th Street (see Fig. 5) and the area between, now one of the finest parts of the city, was a smoky waste of tracks and service buildings; or, unless he compares a photograph of that area as of today with one, say, as of 1906 (see Figs. 6, 7, and 14).

The writer wishes to add a word of appreciation and indebtedness to the author for his vision in planning the enterprise, and for his record of the whys and wherefores as an inspiration to others.

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DISCUSSIONS

EARTHQUAKE STRESSES IN THE SAN FRANCISCO-OAKLAND BAY BRIDGE

Discussion

BY MAURICE A. BIOT, ESQ.

MAURICE A. BIOT,[6] ESQ. (by letter).[6a]—It was found that for a simple structure with one degree of freedom, subjected to a horizontal motion, the maximum shear force due to an earthquake could be expressed as

$$V = m\,A(T) \qquad (55)$$

in which m is the total mass, and A an acceleration function of the period of the structure but characteristic of a given earthquake. This quantity A is referred to as the "equivalent acceleration" for a simple resonator because of the fact that, for a structure of given period T_1, a static acceleration of value $A(T_1)$ produces the same stress as the earthquake on that particular structure. Eq. 55 may be written in a different form as

$$V = V_g \frac{A(T)}{g} \qquad (56)$$

in which V_g represents the maximum shear under a static horizontal force due to gravity.

The equivalent acceleration $A(T)$, plotted as a function of the period, is a characteristic curve of the earthquake called a spectrum. Such spectrum curves were evaluated by the use of a mechanical analyzer for the earthquakes of Helena, Mont., on October 31, 1935, and Ferndale, Calif., on September 11, 1938, for which the records show an acceleration peak of $0.16\,g$ and $0.17\,g$, respectively. It was found that, for periods greater than 0.2 sec, the envelop of both spectra could be approximately represented by the formula

$$A = \frac{0.2\,g}{T} \qquad (57)$$

in which T is the period in seconds. Eq. 56 may be generalized so as to take

NOTE.—This paper by Norman C. Raab, M. Am. Soc. C. E., and Howard C. Wood, Assoc. M. Am. Soc. C. E., was published in October, 1940, *Proceedings*.

[6] California Inst. of Technology, Pasadena, Calif. On leave from Columbia Univ., New York, N. Y.

[6a] Received by the Secretary November 20, 1940.

into consideration structures with more than one degree of freedom. The maximum shear and the bending moments in each mode of vibration excited by the earthquake may be written, respectively,

$$V_n = C_n V_g \frac{A}{g} (T_n) \qquad (58a)$$

and

$$M_n = B_n M_g \frac{A}{g} (T_n) \qquad (58b)$$

in which V_g and M_g are the maximum shear and bending moments produced by a static horizontal force equal to gravity, and T_n is the period of the particular mode considered. The coefficients C_n and B_n may be called "efficiency factors," depending on the type of structure and the order of the excited mode. Values of these coefficients for a cable and a pin-ended uniform truss are given in Table 2. Using the data in the paper, one can apply these results to evaluate

TABLE 2.—Coefficient for a Cable and a Pin-Ended Uniform Truss

Order of excited mode	$n = 1$	$n = 3$	$n = 5$
C_n for cable	0.816	0.0905	0.0326
C_n for truss	0.816	0.010	0.0013
B_n for truss	1.03	0.012	0.0016

an upper limit for the stresses which would be produced by an earthquake of the Helena or Ferndale type in the San Francisco-Oakland Bay Bridge.

Eqs. 55 to 58 apply only where the ground moves rigidly and would not cover the case in which the towers of the bridge and anchorages are moving out of phase. However, in the latter case it may be verified that the stresses are generally less than if the ground were rigid.

Center-Span Truss.—The measured period in the center-span truss being 9.0 sec and the coefficient $C_1 = 0.816$ (from Eqs. 57 and 58*a*), the maximum shear in the fundamental mode is $V_1 = 0.017\ V_g$. This is the same as that produced by a static force of 1.7% of gravity.

Similarly for the bending moment—applying Eq. 58*b* with $B_1 = 1.03$—$M_1 = 0.022\ M_g$, it is possible to state that the equivalent accelerations for this case are 1.7% and 2.2% of gravity.

Side-Span Truss.—According to Table 1, a probable period of the side span is 3 sec. This corresponds to end conditions intermediate between pin-ended and fixed. The maximum shear and bending moment for the fundamental mode are $V_1 = 0.053\ V_g$, and $M_1 = 0.067\ M_g$. The equivalent accelerations for the shear force and bending moments are, respectively, 5.3% and 6.7% of gravity.

Cables.—The periods for the center-span and side-span cables are, respectively, 5.9 and 3.1 sec. Hence, the corresponding maximum shear stresses are the fundamental mode of these cables: $V_1 = 0.027\ V_g$, and $V_1 = 0.053\ V_g$. The equivalent accelerations are 2.7% and 5.3% of gravity.

Higher Modes.—The stresses in the higher modes are smaller than in the fundamental. Consider, for instance, the case of the side-span truss. The excited symmetric mode next to the fundamental will have a period of about 0.3 sec. Using the coefficients $C_3 = 0.010$ and $B_3 = 0.012$ of Table 2, the shear and the bending moment are found to be: $V_3 = 0.0066\ V_g$, and $M_3 = 0.008\ M_g$. These stresses are negligible compared to those in the fundamental mode. A similar conclusion holds for the cables.

Conclusions.—The writer has computed the stresses that the Helena and Ferndale earthquakes would produce in the San Francisco-Oakland Bay Bridge and has found that a stress corresponding to a static force of 6.7% of gravity could be produced in the side-span truss. The peak acceleration of the earthquakes considered is about 17% of gravity. Since stronger earthquakes with a peak intensity of 30% to 40% of gravity are not improbable, it seems that one should consider side-span stresses corresponding to an equivalent acceleration of about 10% to 12% of gravity. It must be remembered, however, that the effect of the damping has been neglected. This effect for large stresses can be quite considerable, due to the friction at the expansion joints, local plastic deformations, and the dissipation of energy by radiation in the ground through the foundation and the anchorage. Further research is necessary before the effect of the damping on earthquake stresses, and various other reducing factors, can be evaluated correctly.

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PROCEEDINGS

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AMERICAN SOCIETY OF CIVIL ENGINEERS

COMING MEETINGS

Board of Direction Meetings

January 13-14, 1941:

A Quarterly Meeting will be held in New York, N. Y.

April 21-22, 1941:

A Quarterly Meeting will be held in Baltimore, Md.

ANNUAL MEETING

NEW YORK, N. Y.

January 15, 16, 17, 18, 1941

SPRING MEETING

BALTIMORE, MD.

April 23, 24, 25, 26, 1941

The Reading Room of the Society is open from 9:00 A.M. to 5:00 P.M. every day, except Saturdays when it is closed at 12:00 A.M. It is closed all day on Sundays and holidays.

Members, particularly those from out of town, are cordially invited to use this room on their visits to New York, to have their mail addressed there, and to utilize it as a place for meeting others. There is an ample file of current periodicals, the latest technical books, and the room is well supplied with writing tables.